Life Cycle Goal Achievement or Portfolio Volatility Reduction?

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Abstract

This paper is concerned with the use of currently available technology to provide individuals, financial advisors and pension fund financial planners with detailed prospective financial plans tailored to an individual's financial goals and obligations. By taking account of all prospective cash flows of an individual, including servicing current liabilities, and simultaneously optimizing prospective spending, saving, asset allocation, tax, insurance, etc. using dynamic stochastic optimization, the paper addresses the question of the title by comparing the results of such a goal-based fully dynamic strategy with representative current best practices of the financial advisory industry. These include piecemeal fixed allocation portfolios for specific goals, target-date retirement funds and fixed real income post-retirement financial products, all using Markowitz mean variance optimization which is based on the very general goal of minimizing portfolio volatility for a specific portfolio expected return over a finite horizon. Making use of the same data and calibrated Monte Carlo stochastic simulation for all the alternative portfolio strategies, we find that flexibility turns out to be of key importance to individuals for both portfolio and spending decisions. The performance of the adaptive dynamic goal-based portfolio strategy is found to be far superior to all the industry’s Markowitz-based approaches. Superiority is measured here by the certainty equivalent increase in expected utility of individual lifetime consumption (gamma) and the extra initial capital required by an individual to put the dominated strategy on the same footing as the optimal dynamic strategy (certainty-equivalent gap). These results should put paid to the commonly held view amongst finance professionals that the extra complexity of holistic dynamic stochastic models is not worth the marginal extra value obtained from their employment. We hope that these strategies implemented in currently available technologies will rapidly find acceptance by individuals, financial advisors and pension funds to the genuine benefit of individual investors.

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1 Disclaimer: The contributions of this author to the paper are made in a personal capacity and do not represent the views of Alexander Forbes SA.

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1. Introduction and Background

[The] assumption that the future will be like the past, at any rate in the economic sphere, is perhaps more questionable now than for decades. All around the world we stand on the cusp of a dramatic shift in the structure of our populations, the aging of our people (Goodhart and Erfuth, 2015).

This long-term demographic problem, coupled with the uncertainties of recovery from the recent global financial crisis and the corresponding inexorable rise of government welfare costs, has led to a dramatic worldwide shift in retirement provision from governments and companies to individuals, by means of defined contribution pension schemes and tax advantaged individual retirement savings. Accompanying this shift has been steadily increasing government regulation of the investment management industry, particularly regarding individual financial advice given to all levels of society.

Financial planning for the benefit of individuals is based on a variety of approaches internationally. These range from simple heuristic approaches for selecting portfolios to approaches incorporating the joint stochastic optimisation of asset allocation, allocation to different savings vehicles and setting saving and withdrawal rates. As affordable computing power continues to increase, bandwidth expands and the efficiency with which we can solve large stochastic problems evolves, this range widens, with ever more complex financial planning tools emerging. As we enter the age of big data, this trend will surely continue.

Despite this relentless march of development, simple heuristic methodologies and mathematical approaches long criticised in the literature continue to enjoy widespread acceptance by the financial planning industry.

An important contributing factor to this divergence of approach is the difficulty of measuring and understanding the incremental benefit of incorporating more of the complexity of the household life cycle financial planning problem. The results of complex stochastic modelling have gained most widespread acceptance by the general public in areas such as meteorological modelling, where it is easy for the man on the street to judge efficacy and benefit.

In recent literature, some progress has been made in measuring the efficacy of different approaches to solving the individual financial planning problem. For example, Blanchett & Kaplan (2013) introduce the notion of gamma, measuring the increase in the certainty equivalent income in moving from a benchmark approach to the approach recommended by an advisor, given certain assumptions about utility and intertemporal discounting. Arguably the notion of gamma may be too abstract to gain popularity amongst clients purchasing financial advice, but the measure has expanded the toolkit available for motivating advisors to refine their advice.

This paper adds to the existing literature by analysing and decomposing the value added by a stochastically optimised holistic goal-based approach to financial planning, reflecting additional functionality not tested by Blanchett and Kaplan. For example, we measure the benefit of incorporating dynamic recourse decision making and test the strategy for individuals' savings toward
retirement and other financial goals. We consider an industry standard mean variance approach to asset allocation, a fixed drawdown in real terms of income post retirement, and saving only in a pension fund. We measure the benefit of incrementally incorporating:

- an optimal asset allocation as informed by mean variance optimization
- varying the level of risk of the mean variance optimal strategy
- selecting a strategy that is optimal with regard to a utility of lifetime income objective
- dynamic strategies that are allowed to vary with time
- recourse decision making
- setting portfolio withdrawal (drawdown) amounts optimally.
- multiple channels or portfolio wrappers with different tax treatments and asset allocation limits
- setting portfolio withdrawal (drawdown) amounts optimally.

We focus our attention on comparisons to techniques widely used in the financial planning industry, rather than to techniques in the current literature competing for acceptance. Our aim is to contribute to the understanding of whether the techniques used by industry are inefficient due to not making use of certain technologies and if so, how large these inefficiencies are. We generalise some features of strategy construction methodologies rather than addressing each approach individually to offer results that can be interpreted more broadly by the industry.

A wide view is taken of what constitutes financial planning for individual benefit, including financial advisors meeting and advising individuals, the decisions of defined contribution trustee boards and products marketed by industry as addressing the lifecycle consumption problem (such as target-dated funds and living annuities. We recognise that these varied sources and influences affect the advice given through constraints and legacy. In particular we recognise that advice is influenced by:

- The sophistication of the entity providing the advice.
- The sophistication of the individual or entity receiving the advice.
- Whether the advice is delivered directly to individuals or to an entity making decisions on behalf of a group of individuals. An example of the latter is a board of trustees of a defined contribution pension fund who receive advice they will use to make decisions on behalf of the members of their pension fund. The cost of accessing advice, economies of scale and how much is known about the individual investor’s collective finances and his or her objectives all play a role here.
- Whether the advice is intended to address the holistic finances of the individual or to offer advice on a component of the individual’s finances (for example saving for a child’s education).
- Legal and regulatory requirements on the provision of advice.
- Whether the advice is pre or post retirement.

For each channel, significant differences exist in the best advice delivered and the manner in which this advice is derived. Irrespective of which of these frameworks or influences are at play, in all cases an entity claims to be an expert advisor, dispensing advice with a view to positively influencing the lifetime consumption of an individual household or many individual households (see US GAO, 2014). There is thus a common purpose in all of these practices.

Section 2 of this paper reviews the basis of contemporary and prospective individual investment planning advice given by industry in more detail and discusses recent attempts to measure the value added by the decision support systems used by financial advisors to an individual. A brief overview
of the iALM financial planning tool that we will use to assess the generic capabilities of the systems described is given in Section 3. In Section 4 the basis of these comparative system experiments is described in terms of the evaluation of value added, the individual profiles involved and the alternative lifetime portfolio allocation strategies considered, as well as the data and methods employed for each. Section 5 contains the comparative empirical results regarding expected objective values and the statistics of goal achievement. Conclusions are drawn in Section 6, which emphasizes our clear empirical demonstration of the superiority and flexibility of employing holistic goal based dynamic stochastic optimization techniques for financial advice. Goal achievement histograms for the alternative portfolio allocation strategies applied to two representative individual profiles are contained in an appendix.

2. Current and Prospective Individual Investment Advice

The literature on optimal investment strategies for retirement and more generally optimal financial planning is vast. For a detailed review of portfolio optimisation and the incorporation of investor liabilities and utility frameworks, consult Thomson (2003). For a review of more recent approaches adopted for financial planning the reader is directed to Dempster & Medova (2011) (see also Medova et al., 2008, and Dempster and Medova, 2012).

2.1 Industry review

The focus of this paper is to identify whether significant inefficiency exists in current market practice, due to a limited uptake of advanced planning tools and technologies. We hence focus on literature summarising techniques widely used by industry rather than on techniques in the current literature competing for acceptance. We would also like to stress that financial advisors add value in a variety of other ways beyond the algorithmic decision making components of their advice; an investigation of these added benefits is beyond the scope of this article.

We summarise the typical bases we have observed being used by the industry globally for financial planning advice, grouping these by key characteristics of the algorithmic component of the advice. The key characteristics which we have used to differentiate strategies include:

1. Does the approach optimise the strategy being used by a member, or simply test a limited number of strategies specified by an advisor?
2. Is there an explicit link to clients’ long term financial goals in optimising the strategy used?
3. If the strategy is evaluated against clients’ long term financial goals, is a simple measure such as shortfall probability used, or is the extent to which a client’s needs have been met considered? An standard example of the latter is the use of expected discounted utility.
4. Does the approach reflect an assumed average member, or does it reflect the circumstances of an individual being advised?
5. Upon retirement, Is a fixed annuitization assumption made, or are living annuities or similar drawdown strategies considered?
6. Where living annuities are considered, are withdrawals and contributions toward savings set dynamically?
7. If withdrawals are set dynamically, are these optimised or determined using simple heuristics?
8. Does the modelling reflect dynamic recourse decision making?
9. Does the approach optimize between different savings vehicles or wrappers, with different tax regimes and investment limitations?
We now group industry practices into generalized categories using these characteristics. Although we make no claim to reconcile the value added by any single specific approach used, we hope that our generalization will offer some insight into a wide range of market practices. Note that many of these approaches only address components of the broader financial planning problem.

**Simple rules or optimisation with no link to objectives or liabilities**

An enormous number of simple approaches exist to setting investment strategies without a direct link to the liabilities of pension fund members or individual clients of financial planners. Perhaps one of the best known theoretical approaches is modern portfolio theory (or mean-variance optimisation), originally developed by Markowitz (1952). Is there evidence that modern portfolio theory is still recognised and used by practitioners?

The Certified Financial Planner Board of Standards, Inc. (CFP Board) established in 1985 sets and enforces the requirements for the CFP® certification in the United States³. At the time of writing, the CFP Board oversees more than 70,000 professionals⁴. CFP Board (2013) describes modern portfolio theory as part of the “foundation for any evidence-based approach to assembling a client portfolio” and asserts that “it is only since the 1952 publication of ‘Portfolio selection’ by Harry Markowitz that the correct way to measure the risk and return of a particular portfolio allocation was established”. There is thus evidence that the approach is still presented to aspiring financial planners in educational material as relevant. There is also evidence in the media that suggests the approach is still being used by financial planners in their advice to individuals on the ground⁵,⁶.

In many cases, mean-variance optimisation (MVO) is used in approaches that purport to be tailored to the financial needs of individuals. For example De Giorgi & Hens (2009) indicate that risk profile questionnaires are typically used to determine an individual’s risk profile. A few master portfolios are selected off the efficient frontier and the final portfolio is based on a mapping between the results of the questionnaire and the master portfolios (op cit.). The quality and relevance of these risk-profile questionnaires and the manner in which the results are mapped into portfolio choices varies widely in practice.

According to the United States Government Accountability Office (GAO, 2014), managed accounts are:

“investment services under which providers make investment decisions for specific participants to allocate their retirement savings among a mix of assets they have determined to be appropriate for the participant based on their personal information”.

Managed accounts attempt to “customize” or “personalize” an investor’s strategy to their circumstances. “Customized” services design strategies based on information easily obtained from a pension plan’s record keeper such as contribution rates, sex, income and the individual’s current account balance. “Personalized” services incorporate additional information such as an individual’s

³ http://www.cfp.net/about-cfp-board/about-cfp-board/history#sthash.NUunamkd.dpuf
⁴ http://www.cfp.net/news-events/research-facts-figures/cfp-professional-demographics
⁵ “the primary model used in practice has been the mean-variance model which is extensively applied in asset allocation and active portfolio management.” Hovey and Wysel (2012) in the Financial Planning Magazine, which is the official publication of the Financial Planning Association of Australia, the professional association for the Australian financial planning profession.
⁶ “Chances are that if you manage investment portfolios, modern portfolio theory (MPT) lays the groundwork for how you do it. It provides a framework for advisors to help their clients find the best return on their money with the least amount of risk.” A Lavine (1 March 2010), Markowitz: MPT Holds Up. Financial Advisor available at http://www.fa-mag.com/news/does-it-hold-up-5153.html
risk preference. Although these strategies include some tailoring to individual needs, the actual underlying strategy is not optimised to these needs. In most cases, the investment strategy is developed using conventional mean-variance optimisation or without any formal optimisation (op cit., pp. 17-18).

Merton (2012) states that “the mean-variance portfolio model is still the core of most professional investment management models, even for sophisticated institutions”, suggesting widespread usage by institutional investors. The authors of this paper are also aware of examples of large pension fund consultancies that use this approach when advising their clients.

Many other simple rules are used in the industry that do not relate to client liabilities. We choose to use modern portfolio theory here as the benchmark strategy from which we will investigate the value of increasingly sophisticated stochastic approaches to designing portfolios for individuals. The choice is arbitrary, other than that we believe the MVO approach remains popular and that the tests will be beneficial to a broad audience.

**Simple stochastic modelling and off-the-shelf portfolio evaluation without optimization**

A large number of tools exist that offer advisors the functionality to test how well a pre-defined strategy will perform in the context of their clients’ objectives. These tools cannot design optimal investment, drawdown, contribution or spending strategies; instead off-the-shelf portfolios and products are tested and analysed with simple, static assumptions about the other components of the strategy. The specific products tested are selected by the advisor, making any attempt at optimisation a manual, iterative process, limited to products available in the market. The quality of the portfolios selected and proposed is hence dependent on the quality of the advisor and perhaps some luck. The portfolios or products the advisor selects from are generally designed by a separate entity, typically an asset manager. The basis used by an asset manager to construct these portfolios can vary significantly, and seldom explicitly considers long term savings objectives for individuals, like securing a stable retirement income. In cases where such objectives are considered, an average member profile must be used by the asset managers.

Generally the liability related outputs from systems such as these are relatively simple. For example, these tools might answer questions such as how long a client can expect to sustain monthly withdrawals at a certain level or what the probability of financial ruin is. Critical to this categorisation is a simple link to liabilities without the sophistication of conventional utility functions or other techniques to measure the depth of shortfall or surplus income achieved. These objectives or liabilities are however set to reflect the specific circumstances of an individual. Typically the modelling itself is also relatively simple and does not reflect dynamic recourse decision making. Recourse decisions are prospective decisions to be made at a future date, based on information that is available to the model prior to the time in the future at which it will be used for advice to a client or to inform a decision.

The framework described by Levitan et al. (2009) is an example of such an approach. The authors can confirm that this paper forms the basis of the approach and tools used by the retail consulting division of one of the ten largest benefit consulting businesses in the world. Under this framework, an advisor assists his or her client to set a required level of income. The approach then dictates testing which of a discrete set of strategies proposed by the advisor results in the lowest probability of ruin. The authors define ‘ruin’ here as outcomes in which the client does not receive an income at least as great as their target income adjusted for the effects of inflation. A range of life and living annuities are considered.
**Target-date portfolios**

A description of target-date funds and the usage of these strategies can be found in Mitchell and Utkus (2012). Target-date funds consist of a series of portfolios each described in terms of an expected year of retirement, typically offered in five-year increments (op cit.). Target-dated funds can be used within pension funds or 401(k) plans, in which decisions are made on behalf of members or participants, or by individuals or financial planners for their client in designing a more bespoke strategy. In the latter arrangement, their choice to use a target-date fund may be based on simple stochastic modelling as described above.

Target-date funds can be used by pension funds to provide default strategies with some link to long term objectives within the institutional environment. By aligning members’ or participants’ expected retirement ages with the nearest dated fund in the target date range on offer, employers and pension funds are able to offer default strategies with some alignment to the long term objectives of a typical individual without incurring the additional cost of individualised advice by financial planners. However, these strategies are not designed to optimally cater for the needs and objectives of each individual. The complexity of the methodologies used to design these strategies varies between providers.

Target-date funds have proved popular; as of 2010, approximately 70% of US defined contribution plans offered target-date funds, and 36% of US defined contribution plan participants were invested (at least partly) in these funds (op cit.). Target-date funds also form the default strategy in the UK’s National Employment Savings Trust (NEST) scheme.

By considering a 2006 sample of US target-date funds, Bodie and Treussard (2007) show that these approaches can reasonably be represented by, and modelled as, a starting allocation of 80% to equities linearly decreasing to a 40% allocation after 40 years. This appears reasonable when compared to the average equity allocation glide paths reported in more recent surveys.

**Optimisation of asset allocations directly against individual objectives using advanced stochastic approaches**

The most sophisticated tools used within the industry make use of advanced stochastic techniques, such as optimising against individualised liability-based objective functions or reflecting recourse decision making. We briefly outline two examples of such systems. These choices of system are arbitrary, other than that both systems are adapted for use in industry and both are described in the literature.

The first example is the Adaptive model described in Fan et al. (2013). The objective function of this approach incorporates the surplus over the amount required to purchase a guaranteed inflation-linked income and a penalty for scenarios where this income is unaffordable. The Adaptive model incorporates recourse decision making, liability targeting and measuring the extent of shortfall. Although the authors stress that annuitization is not assumed, the objective function is framed in terms of the estimated cost of purchasing an annuity to meet the individual’s retirement income requirement. It is interesting to note that although the approach measures the extent of shortfall or surplus, this value is measured in absolute terms rather than in relative terms like surplus or shortfall income.

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A second example is Managed DC described in Merton (2013). This approach aims to maximise the probability of being able to afford a target income post retirement, while ensuring a second, lesser target income is very nearly guaranteed. The methodology hence does target a retirement income objective but again is only concerned with achieving the specified target and does not directly incorporate the size of surpluses or shortfalls into the objective function used. All projections and optimization incorporate individual pension fund member data such as age, salary, gender and current balances. Default targets are set at scheme level and individuals can further customise their target income objectives from the default levels set. The solution can hence be customised to the needs of an individual. The objective function is constructed in terms of the expected cost of securing a guaranteed, inflation-linked income\textsuperscript{11}. Hence, like the Adaptive model, this approach does not necessarily produce an optimal result by which pension fund members or individual clients of advisors could consider alternative annuitization options such as living annuities. It is not clear whether or not recourse decision making is incorporated, as limited detail is provided in the paper about the underlying modelling.

The literature includes examples of techniques to optimise portfolio draw downs (withdrawals) post retirement and to optimise pre-retirement allocations to savings vehicles with different treatments. Blanchett et al. (2012) establish a methodology for setting a post-retirement drawdown rate each year such that the individual has a 75% probability of not outliving their assets. In general, most tools used in industry that incorporate advanced stochastic techniques such as described above assume (implicitly or explicitly) a fixed post retirement strategy, such as complete annuitization at a specific age into a predefined annuity type. Dempster and Medova (2011) jointly optimise for drawdown rates, asset allocations and allocations to savings vehicles with different tax treatments throughout the lifecycle.

Based on the examples we have discussed, it is evident that the industry uses a variety of approaches to inform financial advice. It is also clear that the level of complexity of these approaches varies greatly.

2.2 Value added measurement

Attempts to measure and decompose the value add of objective based financial planning have been made in the recent literature.

Blanchett & Kaplan (2013) point out that the “benefits from 'good' financial planning decisions are often difficult to quantify”. The authors define a new measure of the value a financial planner can add, namely \textit{gamma}. Gamma is defined as:

“the value added by an individual investor making more intelligent financial planning decisions, measured by the percentage increase in certainty-equivalent retirement income over a base case”.

For an individual saving towards retirement, a certainty equivalent retirement income is the fixed level of (real) income which they would be indifferent to choosing over a specified risky retirement income. For example, an individual might be offered the choice between:

\textsuperscript{11} “The income that the Dimensional Managed DC\textsuperscript{®} targets depends on... the prices of hypothetical inflation-adjusted deferred annuity units. Projections for annuity prices depend on interest rates, the inflation outlook and mortality statistics.” W Pfau (2012). Life-cycle Finance and the Dimensional Managed DC\textsuperscript{®} Solution. Advisor Perspectives, 22 May 2012, available at http://us Dimensional.com/media/109948/ITN_Life_Cycle_Finance.pdf
1. An inflation-protected, guaranteed income at retirement of $2000 a month.
2. A strategy that has an equal chance of generating inflation-protected incomes at retirement of $1600 per month or $2700 per month.

If the individual is indifferent between the two options, the first option is said to be the certainty equivalent retirement income of the second risky option. By making assumptions about the utility function of the individual and their inter-temporal preferences for consumption, it is possible to calculate a certainty equivalent retirement income for any well-defined distribution of income outcomes. By mapping strategies to their certainty equivalent income, it is possible to provide a measure of the improvement one strategy offers over another. Direct comparisons of the expected utility for each strategy are generally less informative as the scale and level of expected utility is arbitrary.

On this basis, the authors test the impact for a retiree of moving from a 20% equity allocation and a fixed 4% portfolio draw down after retirement to a “gamma-efficient” strategy. The gamma-efficient strategy generates a certainty-equivalent income 22.6% greater than the base strategy. Expected utility is increased by the same magnitude as an additional return of 1.59% per annum on the base strategy expected portfolio return. This is described as a gamma-equivalent alpha of 1.59%. The relative contributions of each strategy refinement tested are summarised in Table 1.

<table>
<thead>
<tr>
<th>Total Wealth Asset Allocation</th>
<th>Additional Certainty Equivalent Income</th>
<th>Gamma Equivalent Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annuity Allocation</td>
<td>6.43%</td>
<td>0.45%</td>
</tr>
<tr>
<td>Dynamic Withdrawal Strategy</td>
<td>1.44%</td>
<td>0.10%</td>
</tr>
<tr>
<td>Liability Relative Optimization</td>
<td>9.88%</td>
<td>0.70%</td>
</tr>
<tr>
<td>Asset Location and withdrawal sourcing</td>
<td>1.65%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Total</td>
<td>22.63%</td>
<td>1.59%</td>
</tr>
</tbody>
</table>

Table 1. Gamma decomposition determined for various strategy refinements by Blanchett and Kaplan (2013)

It appears that the modelling underlying the authors’ tests did not reflect recourse decision making, based on the description of the simulation done and the characterisation of the utility function used. This limits the conclusions that can be reached from this analysis, as very few strategies will be optimised once in a lifetime for a client or pension fund member. The test case is also limited to a retiree, implying a single financial goal. The inputs or decisions are not jointly optimised in terms of expected utility. For example, the flexible amount withdrawn each year in their dynamic withdrawal is set to ensure a 75% probability of avoiding ruin, i.e. running out of capital before death, rather than to maximize expected utility.

More recently, Das et al. (2014) treats goal-based investment and the value it adds, including the value add of dynamic portfolio strategies. The paper uses a basic, arbitrary objective, maximize wealth subject to expected portfolio shortfall being below a user-specified value. Of course this does not deal with the frequency of shortfalls and treats risk tolerance in a binary fashion with a ‘cut off’ level of tolerance. It should be noted that expected shortfall does not control actual shortfall when it occurs. The author assumes temporal independence of the returns of the two asset classes, stocks and bonds, studied using standard distributions. It is stated that the primary purpose of the paper is to compare static and dynamic rebalancing strategies, not to explore the highest quality objective functions or optimisation techniques. In the sequel we will address the same basic question, but
measuring the improvement with a more realistic, broadly accepted objective, a broader range of alternatives tested and a more granular approach to decomposing value added. Although Asher’s paper is written in terms of only two completely hypothetical asset classes with no link to the real world or market practice, and thus unfortunately can be expected to have little or no traction with industry, in our opinion it represents a good beginning in the right direction.

3. The iALM Financial Planning Tool

The iALM system is based a meta-model of the household life cycle financial planning problem described in Dempster & Medova (2011). See also Medova et al. (2008) for a description of iALM for the US jurisdiction and Medova and Dempster (2012) for a case study using the UK system of a four person UK household through the 2008 financial crisis. Mortality, illness and the need for long term care, as well as various other potential liabilities, are modelled stochastically for individuals whose household finances are being optimised relative to transactions costs. The model also incorporates savings vehicles with different tax treatments and different asset allocations and usage limits -- such as the US 401k account and the UK ISA (Individual Savings Account); savings without any tax shield; and a conventional pension fund. Asset returns at strategic (index) levels and economic variables, such as interest and inflation rates, are also modelled stochastically.

In optimising their decisions about the next investment period, based on current information, the individual’s future decisions are considered over the life cycle. This is referred to as making decisions with dynamic recourse, i.e. decisions that can be adjusted as prospective time evolves and future information becomes available in the problem. This feature of the model results in a tree-like branching structure for stochastic simulations of the prospective future, with each node branching off many times to reflect the lack of certainty faced when making a prospective decision at the corresponding prospective future date. (All the information upon which the model is based will typically be updated for the subsequent actual investment decision.) In optimising the individual’s strategy for the next period, the system thus needs to optimize their decisions at each node in the future along the simulated scenarios. This results in what has classically been referred to as the “curse of dimensionality”, requiring the solution of a large scale stochastic optimization problem. For example, an individual aged 25 who is saving toward retirement, a tertiary education for two children and a home could expect to be optimising over constraints containing more than 2 million variables and 20 million terms, which demonstrates the complexity of the problem financial advisors often attempt to address using simple heuristic methods.

In deriving a solution, the system jointly optimizes:

- The amount an individual should spend each year
- The allocation to each savings vehicle
- The asset allocation required within each savings vehicle
- The amount an individual has available to annuitize post retirement
- The amount post retirement to withdraw from savings each year.

Specifically, this optimization maximizes the expected utility of the individual’s multiple spending goal achievements over lifetime. The calibration of the actual utility curve used can be set in consultation with a financial planner to reflect an individual’s saving and living expense goal preferences or set to a default level for use in pension funds. The usual smooth utility curve is replaced for ease of individual understanding and tractability by a goal-based piecewise linear one. The solution strategy can be further tailored to incorporate meeting additional individual target goals such as education for children, purchasing a home or a once in a lifetime holiday.

All the features that we have listed above as differentiators of advice bases are incorporated in the iALM system, which implies that it can be used to test the relative importance of each feature. We
hope that the definitive results of the tests reported in this paper will encourage the use of such advanced technologies by financial planners, irrespective of the software package that is actually used to implement them.

4. Comparative Experiments

In the next section we will use the UK iALM system to compare several investment strategies applied to variations on two simple individual profiles -- a young working person with no capital and a just-retired person with substantial saved capital for retirement -- as follows:

1. A static asset allocation to a fixed Markowitz buy-and-hold allocation from modern portfolio optimization (MVO) theory. A fixed savings drawdown is assumed to be used after retirement.

2. The assumption of a fixed post-retirement income withdrawal is relaxed, allowing the system to optimize withdrawals in each prospective period.

3. A static asset allocation optimised to expected utility, i.e. one step more complexity and ‘correctness’ than MVO theory using an optimized fixed allocation.

4. A dynamic but non-adaptive asset allocation which in each time period is the same allocation in all prospective scenarios but can vary over time.

5. The full iALM solution to show the effects of the final step to the flexible dynamic recourse solution reacting to all prospective circumstances.

All the above experiments have been implemented using the flexible full functionality of the iALM system. This allows specification for these experiments of some or all of fixed asset allocations, fixed spending/consumption or fixed savings withdrawals.

It should be noted that the fixed life-stage portfolio allocation strategies discussed in the previous section lie between strategies 3. and 4., depending on the flexibility of allowed portfolio adjustments up to the specified problem horizon. Hence, although not specifically tested here, their results can be expected to lie between those of the strategies tested here.\(^{12}\)

We have chosen not to investigate the value added by optimizing the timing and allocation to different annuity types, although this is certainly possible with the iALM system. Instead, we assume a living annuity (i.e. a discretionary draw down facility) is chosen, but the system is used to optimize prospective withdrawals.

In this section, the necessary preliminaries to the experiments will be described. Namely, the appropriate modification of the gamma concept for the full iALM model, the description of the household profiles employed, the input data description and the detailed specification of the above incrementally flexible portfolio allocation strategies relative to individual saving and spending goals.

4.1 Value added evaluation

We begin with a description of the iALM objective function.

**Goal oriented lifetime utility**

Dempster and Medova (2011) state that:

> “The overall objective of the iALM optimization (in today’s value terms) is to maximize the expected utility of lifetime consumption, taking into account total tax payments and excess [unsecured] borrowing.”

\(^{12}\) Indeed, see Figures 7 and 9 below and the discussion in the next section of non-adaptive dynamic strategies.
Note that utility is measured from consumption, adjusted for the effects of inflation, but not by any further intertemporal discounting.

The literature contains many works based on expected discounted utility models and this model has become a generally accepted approach (Frederick et al., 2002). Conventional wisdom says that a utility analysis of available choices requires an instantaneous utility function (much like that used here for each year’s goal spending) applied to real levels of consumption and a discount function to account for peoples’ myopic approach to evaluating consumption options. People generally penalize consumption the further out it is into the future. However, which size and temporal shape of the discount rate to use is a massively contentious issue, and empirical studies have shown evidence for problem results differing by orders of magnitude as this rate is varied (op cit.). Although the discounted utility model does not really fit human behaviour, it is nevertheless still generally accepted and used as a normative model (op cit.). Often a long end gilt yield is used to answer the anguished question of what discount rate it is best to use. Although it is arguable that the discount rate should be higher than this, such a rate is used by a number of authors and, critically, is objective and market determined. However, this ambiguity in rate determination requires that its effects on results must be tested by varying it as a parameter. Unfortunately, as one might expect and was noted above, the types of results of interest here are often quite sensitive to the actual rate chosen, e.g. an optimal drawdown strategy post retirement will be highly sensitive to the discount rate. Moreover, the discount factors used in practice are usually not stochastic, as they stem from human interpretation/reaction to uncertainty rather than reflecting actual economic or life span stochasticity.

In the iALM model an inflation rate stochastic process is used to state all future monetary variables in today’s money. Rather than discounting cash flows back to the present with all its open issues, a variety of inflation processes appropriate to specific costs, such as health care spending, school and university fees, etc., are employed to represent prospective future costs. The resulting costs in future price levels are then discounted back to the present using the general price level inflation process on each path of the scenario tree. The reader should note that expected utility is used here without intertemporal discounting, i.e. assuming no myopic consumption behaviour, due to the complex issues involved in choosing a discount rate. Although it could be argued that the objective approach used here is particularly appropriate to retirement planning with any model that takes account of stochastic life expectancy, any specific discount rate can easily be incorporated in the iALM model and system. Subjective post retirement intertemporal preferences are mainly based on life expectancy however, and it should also be noted that using inappropriate intertemporal discount factors for advice could possibly result in a misrepresentation of the problem and even an increased probability of actual ruin (Frederick et al., 2002). We shall return to this point in Section 5.

Mathematically, the overall objective of the iALM optimization is to maximize the expected utility of lifetime consumption, taking into account different penalties (tax payments, borrowing), i.e.

$$E \sum_{t=1}^{T} 1_{(\text{alive},t)} u_t(C_t),$$  \hspace{1cm} (1)

where

$$u_t(C_t) = u_t(y_t) - (\pi^{xz} z_t + \pi^{T} I_t^f) / \varphi_t.$$

$$u_t(y_t) = \begin{cases} 
0 & \text{if } y_t \leq 0 \\
1 & \text{if } y_t > 0 
\end{cases}$$

\text{where}  \hspace{1cm} 13

\text{We use boldface type to denote stochastic (i.e. scenario dependent) entities.}
Here, at time $t$, $1_{(\text{alive},t)}$ denotes a random 0-1 indicator function to handle the random length of life scenarios, $u_t$ utility, $y_t$ the amount (in current pounds) of income spent, $\varphi_t$ the inflation index, $z_{xt}$ excess (unsecured) borrowing - an auxiliary variable introduced to deal with possible bankruptcy and $I_t^\varphi$ total tax payable, with $\pi^u$ and $\pi^\varphi$ being their respective positive penalty coefficients to ensure that their optimal levels are minimal. The absolute problem horizon $T$ is chosen to reflect the highest age in the life table used in the model.

By excess here, we mean in excess of the individual’s total assets at the time. We are assuming that the investor has a preference for consumption when solvent rather than insolvent, because of the significant negative financial and social impact of being declared bankrupt. The penalty for excess borrowing in (2) prevents solutions that require very large unsecured borrowing positions (although small unsecured borrowings are allowed) as such solutions would be likely to render their advice unacceptable to clients. The tax penalty minimization assumes that an individual prefers to pay less tax than more, all else being equal, i.e. given two different portfolio allocation strategies that generate the same pre-tax revenue over time the one leading to lower taxes would be preferred.

For the profiles we consider here, in each year $t$ there is only one goal, total annual spend on living, but in general there will be several goals each year. The annual spend on each of these goals must be specified by the individual in terms of three intuitive levels: desirable, acceptable and minimum. The utility for each goal in year in each year $t$ is specified by a concave piecewise linear function whose kinks are at these three levels. Only the minimum spend is a hard constraint. As a result, when all the optimal spending variables are close to their acceptable levels or beyond, the objective function value can be interpreted approximately as the expected total lifetime spend by the individual (household). However, when an individual’s goals are far from achievable, the objective function value of the optimization problem will be negative (see Table 4 below).

For simplicity, no allowance is made here for bequest goals, but these are permissible in the iALM system.

**Gamma definition**

Following Blanchett and Kaplan (2013), gamma measures the additional value created by constructing an optimal portfolio relative to an investor’s objective. This is done by comparing the increase in certainty equivalent income when improving an investment or income drawdown strategy. We now present a mathematical definition of this concept.

For the case that the utility of spending is constant (in real terms) through time, the objective of a specified certainty-equivalent lifetime spend of $C$ is defined as a maximum over total lifetime spend $y$ given by

$$u_{ce}(C) = \max \left\{ \sum_{t=1}^T p(\text{alive},t)u_t(y) : \sum_{t=1}^T p(\text{alive},t)y \leq C \right\} . \quad (3)$$

Where $p(\text{alive},t)$ denotes the (life table) probability that the individual is alive in year $t$ which is a non-decreasing function of $C$. Denoting the optimized value of the objective function of (1) by $u^*$, the certainty-equivalent lifetime spend is given by a minimization over $C$ as

$$s(u^*) = \min \left\{ C : u_{ce}(C) \geq u^* \right\} . \quad (4)$$
Now if we have two profiles with the same objective function (1), but with different constraints on investment or spending, then the optimized objective values of the two corresponding problems will be different: \( u^* \) and \( v^* \), respectively. We can calculate the gamma value of the second profile with respect to the first as

\[
\gamma := \frac{s(v^*) - s(u^*)}{s(u^*)}.
\]

The average extra portfolio return required to generate an additional spend of \( s(v^*) - s(u^*) \) over a spend of \( s(u^*) \) is termed the gamma equivalent alpha.

The value \( s(u^*) / L = u_{ce}^{-1}(u^*) / L \), where \( L \) is life expectancy, is the certainty-equivalent spending per annum. If all yearly utility functions \( u_t \) are equal to \( u_1 \) (such as for the retired Profile B we consider in the sequel)

\[
u_{ce}(C) = Lu_1(C/L),
\]

Then the value \( s(u^*) / L = u_{ce}^{-1}(u^*) / L = u_1^{-1}(u^* / L) \).

In the simple profiles studied here, it may nevertheless be the case that the utility of spending changes at the retirement to result in an objective function of the form

\[
E \left( \sum_{t=1}^{T_{ret}} 1_{(alive,t)} u^{emp}(C_t) + \sum_{t=T_{ret}+1}^{T} 1_{(alive,t)} u^{ret}(C_t) \right),
\]

where \( T_{ret} \) is the time of retirement and \( u^{emp}, u^{ret} \) are utility functions of spending pre- and post-retirement, respectively. In this case, certainty-equivalent lifetime spending becomes

\[
u_{ce}(C) = \max \left\{ \sum_{t=1}^{T_{ret}} p(\text{alive},t) u^{emp}(y) + \sum_{t=T_{ret}+1}^{T} p(\text{alive},t) u^{ret}(z): \right.

\left. \sum_{t=1}^{T_{ret}} p(\text{alive},t) y + \sum_{t=T_{ret}+1}^{T} p(\text{alive},t) z \leq C \right\}.
\]

Gamma calculation in this case does not change from (5). However, there can be 2 certainty-equivalent levels of spending, for pre- and post-retirement, defined as the values of \( y \) and \( z \) that give the value \( u^* \) in (4). There may be many pairs \((y, z)\) that correspond to \( u^* \). In that case we choose the pair \((y^*, z^*)\) minimizing \(|y - z|\).

**Certainty equivalent initial gap**

We also report an additional comparison measure of the value added by a new portfolio strategy to the gamma measure. It is specified by the extra initial capital required by the first strategy to yield

---

\(^{14}\) For a general household profile the utility function may change each year as the several spending goals active for that year change, for example when a house mortgage is entered.
the same expected lifetime utility as the new strategy and termed the certainty equivalent initial gap. Consider the objective function (1) used for a specific constrained optimization problem and let the constraint set over which we optimize be denoted by $\mathcal{S}$. The set $\mathcal{S}$ depends on the initial portfolio value $B$, $\mathcal{S} = \mathcal{S}(B)$ and we write

$$v^*(B) = \max_{\mathcal{S}(B)} E \sum_{t=1}^{T} 1_{\{\text{alive}_t\}} u_t(C_t) .$$

Suppose we have another profile with a tighter set of constraints represented by the constraint set $\tilde{\mathcal{S}}$ for which

$$u^*(B) = \max_{\tilde{\mathcal{S}}(B)} E \sum_{t=1}^{T} 1_{\{\text{alive}_t\}} u_t(C_t) \leq v^*(B) .$$

Obviously, $u^*(B)$ is a non-decreasing function of $B$ which may be shown to be concave, see Figure 1. The smallest solution $\Delta B$ of the equation

$$u^*(B + \Delta B) = v^*(B)$$

is the certainty-equivalent initial gap.

### 4.2 Household profiles

Our experiments aim to compare different solutions of the individual asset liability management problem. For these experiments two simple UK profiles were chosen. Basically, a young individual and a retired individual, both of which are single. We shall refer to these as Profiles A and B respectively.

![Optimal expected utility as function of starting wealth](image)

**Figure 1.** Optimal expected lifetime utility as a function of initial portfolio wealth
Profile A
The individual is 30 years old, has no savings, earns £60k gross (equal to about 45k after tax) and has spending goals for (minimum, acceptable and desirable) amounts corresponding to (30k, 40k, 50k) pre-retirement and to (7.5k, 40k, 70k) upon planned retirement at 65.

Profile B
The individual is 65 years old, has just retired and therefore does not earn a salary. He has £600k in initial savings, and his post-retirement spending goals for (minimum, acceptable and desirable) amounts correspond to (7.5k, 40k, 70k).

We will examine three types of solutions for these profiles:
- solutions with various static asset allocations fixed from the beginning and only spending decisions being optimized
- solutions with fixed spending levels and only investment decisions being optimized
- a fully dynamic solution with both investment and spending decisions being optimized.

4.3 Lifetime portfolio allocation strategies

Strategic portfolio assets
For all experiments these are as follows:

domeq - domestic equity. Calibrated on FTSE 100 +2% annual dividend.
inteq - international equity. Calibrated on FTSE World ex UK + 2.5% annual dividends.
corppaa - AA corporate bonds. Calibrated on iBoxx £ Corporates AA.
long - long-term gilts. Calibrated on iBoxx £ Gilts 10+.
com - commodities. Calibrated on S&P GSCI Commodity Total Return converted to pounds sterling.
alt - alternative investments. Calibrated on Credit Suisse Hedge Fund index.
prop - property investments. Calibrated on House price index national monthly growth series from Acadametrics.
tcash - taxable cash investment. Calibrated on 3 month UK Treasury bills.

Arguably, it is more common to use a small number of asset classes (such as a risky and a risk free asset, cf. Das et al., 2014) and a very simple distribution to make replication of results easier for others. While this approach certainly has its place in the literature, a core aim in this paper is to demonstrate the value that can be unlocked by practitioners. We therefore sacrifice some simplicity and tractability in favour of results to which we feel the financial planning industry can relate more easily. We also believe that a number of realistic assets is more reflective of the value that a real world investor might actually gain from using each of the technologies tested here.
For each of the 8 series above, 10 years of monthly data to end 2013 has been used for calibration. Summary statistics for them are shown in Table 2 to one or two decimal places.

<table>
<thead>
<tr>
<th></th>
<th>Domeq</th>
<th>ineq</th>
<th>corpaa</th>
<th>Long</th>
<th>Com</th>
<th>Alt</th>
<th>prop</th>
<th>tcash</th>
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</thead>
<tbody>
<tr>
<td>expected return</td>
<td>8.3%</td>
<td>11.7%</td>
<td>3.2%</td>
<td>5.4%</td>
<td>4.9%</td>
<td>8.2%</td>
<td>4.0%</td>
<td>0.4%</td>
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<td>st.dev</td>
<td>14.5%</td>
<td>16.4%</td>
<td>5.5%</td>
<td>8.8%</td>
<td>23.3%</td>
<td>9.1%</td>
<td>2.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.57</td>
<td>0.72</td>
<td>0.58</td>
<td>0.62</td>
<td>0.21</td>
<td>0.89</td>
<td>1.48</td>
<td>8.62</td>
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</table>

The correlations below correspond to the variances and expectations in the report.

<table>
<thead>
<tr>
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<th>ineq</th>
<th>corpaa</th>
<th>Long</th>
<th>com</th>
<th>alt</th>
<th>prop</th>
<th>tcash</th>
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</thead>
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<td>0.55</td>
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<td>0.18</td>
<td>-0.15</td>
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<tr>
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<td>0.01</td>
<td>0.01</td>
<td>0.12</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long</td>
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<td>-0.21</td>
<td>-0.05</td>
<td>-0.15</td>
<td>-0.29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>com</td>
<td>1.00</td>
<td>0.26</td>
<td>0.22</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt</td>
<td>1.00</td>
<td>-0.05</td>
<td>-0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prop</td>
<td>1.00</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Summary of annual asset return characteristics

Some caveats about the data used are in order. The purpose of this paper is however to demonstrate that:

- significant structural differences in asset allocation result when introducing the latest modelling techniques, and
- the value add of these technologies is significant for investors.

An investigation into the nature and sensitivity of these superior techniques to changes in modelling assumptions must be left to further research.

First note that the data period from 2003 to 2013 spans the pre-crisis bubble, crash and limited recovery, which affects the various asset class returns differently over time. In particular, bonds have seen one of the strongest bull markets to date, largely due to US, UK and Japanese monetary policy over the sample period. The resultant expected return for long bonds is probably unrealistically high in light of current US and UK treasury/gilt curves, particularly post tapering, and in the model for use over a lifetime horizon the return must be based on realistic assumptions of the timing of its return to normal levels. Long bond return is also very high relative to shorter bonds and cash, suggesting an unrealistically steep yield curve or significant further reductions in yield, which are currently unlikely to continue into the future in the UK.

Regarding property returns, it should be noted that these are not the returns and volatility of a client’s individual home(s); in the iALM model, for more complex profiles than are used here, such returns are given by a property inflation rate driven by the general price level inflation rate process. The asset class used here has a very low volatility and low correlation with other assets in the modelling, since it is represented by a national house price index stemming from a diversified
portfolio of houses. It is meant to represent the continuing UK trend of investment in buy-to-let residential property and appears to be essentially the perfect asset. However, the average buy-to-let investor will actually face the idiosyncratic features of their specific portfolio. If they in fact hold a widely diversified property portfolio their returns will have a tendency to stronger correlation with the rest of the market, a lower return and likely a far higher volatility.

Finally, we note that asset returns are not necessarily assumed to be independent across time periods (here years) due to the diffusion asset return models used in iALM, see Dempster and Medova (2011) for details. This is non-trivial in that:

- A lack of independence makes the problem solution significantly more complex.
- To the authors’ knowledge many of the asset return models used in the industry do not assume independence of the distributions of returns in subsequent periods. Hence we demonstrate that strong views on non-independence of returns can be accommodated when using the techniques we explore here. This is particularly important at a time when regulators and practitioners around the world are acutely sensitive to the importance of prudent approaches to the evolution of volatility processes in stochastic modelling.

For indices at a monthly time step it can be argued that normal (Gaussian) shocks are a reasonable specification and that in any event the effects of alternative fat tailed monthly distributions tend to average out for annual portfolio decisions.

While doubtless myriad improvements in the underlying models are possible, we hope that the reader will focus on the material comparative results achieved in the empirical section in the sequel.

Markowitz efficient frontier

The efficient frontier for Markowitz mean variance optimization portfolios with deterministic allocations and short-sale restrictions has been calculated for a one year horizon using the asset return statistics in Table 2. For the fixed portfolio asset allocation models, we consider fixed-mix portfolios (Dempster et al., 2011), i.e. portfolios rebalanced annually to the same fixed asset allocations, over long term horizons. In this case long-term efficient portfolios are very similar to one-year efficient portfolios. Any differences in results for varying horizons for standard MVO model simulations are caused here by the fact that some of the return processes actually start from non-equilibrium states, e.g. long bonds, as noted above.

The efficient frontier was constructed from the asset class expected returns, volatilities and correlations of Table 2 used in the iALM system. The calculated efficient frontier and the proportional allocations in portfolios on the efficient frontier are shown in Figures 2 and 3 respectively.
Figure 2. Portfolio return volatility (x axis) mean (y axis) efficient frontier

Asset allocations on the efficient frontier

Figure 3. Proportional portfolio allocation evolution along the frontier

Statistics for the three standard deterministic MVO portfolio strategies chosen to be representative of commonly used degrees of individual profile risk aversion are shown in Table 3.

<table>
<thead>
<tr>
<th>Portfolio characteristic</th>
<th>conservative</th>
<th>moderate</th>
<th>Aggressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected return</td>
<td>4.5%</td>
<td>7.8%</td>
<td>11.0%</td>
</tr>
<tr>
<td>volatility (st.dev.)</td>
<td>2.3%</td>
<td>6.6%</td>
<td>14.1%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>1.92</td>
<td>1.17</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Table 3. Summary of annual characteristics of 3 MVO portfolio strategies
Static portfolio asset allocation strategies

The static strategies have proportional asset allocations kept fixed throughout life. A static strategy is implemented in the model as an additional asset - a mutual fund with the desired characteristics and all an individual's assets are constrained to be in that mutual fund. Due to annual rebalancing to fixed asset allocations over long term horizons, some proportion of the reported returns achieved with these strategies represent a rebalancing dividend (see Dempster et al., 2011).

For example, in a two period model with a thousand simulations at each node of the decision tree, only one asset allocation would be modelled. We develop this example through our remaining strategy definitions.

Non-adaptive dynamic strategy

The dynamic but not path-dependent, i.e. non-adaptive, solution adjusts portfolio asset proportions annually independent of specific scenario realizations. It depends only on the averages of decision variables across scenarios and was implemented by suitably modifying the iALM model. Since decisions on asset allocations in the iALM model are made in terms of quantities, the quantities of assets owned are thus the same across all scenarios.

In a two period model with a thousand simulations at each node of the decision tree, two asset allocations would be modelled, one for the first period and one for the second.

Fixed spending

For the young individual of Profile A, the fixed spending strategy is taken to be spending a constant amount of money (after inflation-adjustment) every year before retirement. The level of individual spending was chosen to coincide with the fully dynamic strategy’s average optimal spending before retirement. This of course tilts the comparative results in favour of the fixed strategy, i.e. the level chosen is being informed by a vastly more sophisticated approach. However, applying the same principal to the young individual post-retirement led to bankruptcy in some scenarios. As a result, we compromise by only applying a fixed allocation before retirement and retaining the full dynamic solution after retirement.

For the retired individual of Profile B, a fixed spending strategy is defined as spending a constant amount of money every year in real terms (i.e. after inflation-adjustment of actual spending). This corresponds to the various inflation-index-adjusted "constant" provision products available in the market and often recommended by advisors.

There are also conceptual difficulties in selecting such a constant amount for our experiments with the fixed spending strategy. For this purpose for the retired individual we chose to employ the highest level of spending that can be achieved in our model without having to resort to borrowing. The size of the difference made by a dynamic portfolio drawdown strategy post retirement is perhaps the least well understood aspect of this problem by far both in industry and probably the literature as well.

The fixed drawdown rate is set here as the maximum drawdown that gives a 0% probability of ruin, i.e. of having nothing left to spend across all scenarios. We appreciate that this is a far more conservative approach than is typically used in industry, but there is very little by way of transparent objective approaches to setting these rates in practice. Some firms will use a set of drawdown rates which it would not be prudent for clients to exceed, but which is only recommended to their consultants. These rates are often set to correspond to a 25 to 30% ruin probability that a client’s real income cannot be sustained over expected lifetime. However, the occurrence of this unpleasant situation does not necessarily imply complete bankruptcy as portfolio drawdowns may be capped by law, for example in Canada and South Africa.

15 Using a patented technique.
For more on drawdown setting practice the reader is referred to the US Society of Actuaries report (MacDonald et al., 2013), which is quite comprehensive, particularly on whether or not annuitization is beneficial. The report provides a reasonably comprehensive survey on setting drawdown rates. What is immediately clear from the report’s in-depth review of the literature is the lack of consensus regarding the exact methodology to be chosen or even how to specify the general basis for this problem’s solution. Specifically regarding drawdown strategies the report says that:

“clear, unambiguous, and disinterested guidance on how best to draw down individual retirement accounts and manage the large associated risks has not been widely disseminated. In large part, individuals have been left to decipher conflicting and potentially self-serving advice from financial advisors, or to follow social norms that may or may not fit their personal circumstances and objectives”

Setting a specific drawdown limit will have a large impact on sustainability results. The approach used here of setting the income drawn to the highest sustainable level for the retired profile (B) is more prudent than many of the approaches employed in practice. It might be argued that people actually draw down far more than would yield the 0% risk we are examining here, in order to run the perceived low risk of running out of assets in favour of a considerably higher utility in most real world outcomes.

**Dynamic strategy**

For this strategy we run the full iALM system for the two profiles A and B and report the results below.

In a two period model with a thousand simulations at each node of the decision tree, a thousand and one asset allocations would be modelled, one for the first period and one thousand for the second period. Each of the thousand allocations made at the beginning of the second period would represent the unique decision that would be made following the outcome of one of the one thousand unique simulations of the returns in the first period.

**5. Results and Analysis**

Our experiments decompose the value added by optimizing the optimal expected value of lifetime utility with a fully dynamic strategy more granularly. In moving from the fixed strategies to the optimised strategies we are combining a number of incremental strategy value adds:

1. The effect of switching from a fixed MVO strategy to a liability/utility relative fixed strategy optimisation

2. The effect of dynamism (allowing a strategy that varies across time)

3. The effect of path dependent dynamism (allowing a different strategy depending on the known path up the point of the decision).

Each of those steps adds complexity to the problem to be solved. As a result all current solutions used in practice ignore one or more of these features which makes the problem easier to solve. The impact of each of these effects is very poorly understood by practitioners and they are often dismissed as unnecessary complexity (or dubbed ‘spurious’), even by modelling experts. Such dismissals are usually not based on any evidence, but they do contribute to why the holistic features of iALM, or an equivalent approach, are perceived as unnecessarly. We hope that the results in this section will show the practical importance of the advanced features employed in the reported experiments.
**Dynamic life cycle goal oriented portfolio strategy versus static buy and hold**

We can assess the value-added to the fixed MVO allocation by dynamic optimisation by comparing the optimal values of the objective function for the fully dynamic model with those of the alternatives. However, these values are difficult to interpret due to the arbitrary level and scaling of utility. We therefore report the two different simple measures of Section 4 to assess the dynamic value add – *gamma* and the *certainty-equivalent gap* – as well as other comparative statistics of interest. The results are summarised in the tables below.

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<thead>
<tr>
<th></th>
<th>Profile A</th>
<th>Profile B</th>
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</thead>
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<tr>
<td>Dynamic</td>
<td>1,997,466</td>
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<tr>
<td>Fixed allocation_Conservative</td>
<td>-886,506</td>
<td>-865,730</td>
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<td>Fixed allocation_Moderate</td>
<td>348,687</td>
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<td>Fixed allocation_Aggressive</td>
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</tr>
<tr>
<td>Fixed spending</td>
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</table>

**Table 4. Objective function values**

<table>
<thead>
<tr>
<th></th>
<th>Profile A pre-ret</th>
<th>Profile A post-ret</th>
<th>Profile B</th>
</tr>
</thead>
<tbody>
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<td>Dynamic</td>
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<td>Fixed allocation_Conservative</td>
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<td>Fixed allocation_Aggressive</td>
<td>38,290</td>
<td>41,155</td>
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<tr>
<td>Fixed Spending</td>
<td>39,455 (set)</td>
<td>42,053</td>
<td>29,001</td>
</tr>
</tbody>
</table>

**Table 5. Average level of pre- and post-retirement spending per annum**

<table>
<thead>
<tr>
<th></th>
<th>Profile A pre-ret</th>
<th>Profile A post-ret</th>
<th>Profile B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>39k</td>
<td>62k</td>
<td>45k</td>
</tr>
<tr>
<td>Fixed allocation_Conservative</td>
<td>37k</td>
<td>9k</td>
<td>24k</td>
</tr>
<tr>
<td>Fixed allocation_Moderate</td>
<td>37k</td>
<td>30k</td>
<td>34k</td>
</tr>
<tr>
<td>Fixed allocation_Aggressive</td>
<td>37k</td>
<td>41k</td>
<td>41k</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>39k</td>
<td>42k</td>
<td>29k</td>
</tr>
</tbody>
</table>

**Table 6. Median level of pre- and post-retirement spending per annum (approximate)**

Looking at the income outcomes in Tables 5 and 6, it isn’t immediately obvious that one strategy is vastly more efficient than another. However, once the simulated outcomes are transformed into utility space large gains are evident in Table 4. This shows that achieving true efficiency is not as easy as simply visually inspecting a chart and making a seat-of-the-pants call, as most advisors lacking advanced tools are ultimately forced to do.
As can be seen from Tables 4 to 6, mis-specifying the optimal risk-return characteristics of the fixed MVO portfolio is equivalent to suffering very considerable losses to an investor’s lifetime wealth. Although, as explained above, fixed spending levels are set where possible to the average spend of the optimal dynamic solution, even the best MVO portfolio is still much worse post-retirement than the optimal dynamic strategy, particularly for the retired individual with significant capital.

The value provided by optimizing the spending decisions is also very significant after retirement. Optimizing spending decisions up to retirement is obviously not as important as doing so post-retirement (because the individual always has his salary to rely on) but in practice it’s actual impact can still be considerable. This is of course not the case here as we have set fixed spending per annum for the young profile at the optimized average. In this regard, we should mention that setting a higher fixed spending goal before retirement can lead to very considerable under-saving before retirement, and therefore to under-spending after retirement. So the use of the optimal dynamic spending solution in this case also plays a role in yielding similar effects post-retirement to those of the retired individual.

Value add calculations for the young and retired profiles

The results of the gamma calculation relative to the dynamic strategy and other certainty-equivalent quantities for the young and old profiles reflect similar findings.

<table>
<thead>
<tr>
<th></th>
<th>c-e spending over life</th>
<th>annual c-e spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>1,997,366</td>
<td>41,583</td>
</tr>
<tr>
<td>Fixed allocation_Conservative</td>
<td>1,453,344</td>
<td>30,257</td>
</tr>
<tr>
<td>Fixed allocation_Moderate</td>
<td>1,659,209</td>
<td>34,543</td>
</tr>
<tr>
<td>Fixed allocation_Aggressive</td>
<td>1,818,123</td>
<td>37,851</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>1,896,408</td>
<td>39,481</td>
</tr>
</tbody>
</table>

Table 7. Certainty-equivalent quantities for the young profile

<table>
<thead>
<tr>
<th></th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>0.00%</td>
</tr>
<tr>
<td>Fixed allocation_Conservative</td>
<td>37.43%</td>
</tr>
<tr>
<td>Fixed allocation_Moderate</td>
<td>20.38%</td>
</tr>
<tr>
<td>Fixed allocation_Aggressive</td>
<td>9.86%</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>5.32%</td>
</tr>
</tbody>
</table>

Table 8. Gammas of the dynamic strategy with respect to constrained strategies for the young profile
Table 9. Certainty-equivalent quantities for the retired profile

<table>
<thead>
<tr>
<th></th>
<th>C-e spending over life</th>
<th>C-e spending per annum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>776,055</td>
<td>38,949</td>
</tr>
<tr>
<td>Fixed allocation_Conservative</td>
<td>519,878</td>
<td>26,092</td>
</tr>
<tr>
<td>Fixed allocation_Moderate</td>
<td>647,155</td>
<td>32,480</td>
</tr>
<tr>
<td>Fixed allocation_Aggressive</td>
<td>721,629</td>
<td>36,217</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>614,387</td>
<td>30,835</td>
</tr>
</tbody>
</table>

Table 10. Gammas of the dynamic strategy with respect to constrained strategies for the retired profile

<table>
<thead>
<tr>
<th></th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>0%</td>
</tr>
<tr>
<td>Fixed allocation_Conservative</td>
<td>49%</td>
</tr>
<tr>
<td>Fixed allocation_Moderate</td>
<td>20%</td>
</tr>
<tr>
<td>Fixed allocation_Aggressive</td>
<td>8%</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>26%</td>
</tr>
</tbody>
</table>

Table 11. Certainty-equivalent gap to the dynamic solution

Table 11 shows, for example, that the young individual employing a conservative fixed MVO portfolio strategy would need an initial windfall of 1.5 million pounds to expect to achieve the same utility of lifetime consumption with it as the alternative dynamic strategy would yield with no initial capital.

Many practitioners will concede that liability optimised approaches make sense, but that such a complex analysis is unnecessary because they believe that investors should be in a highly aggressive strategy which will in the long run deliver the best results, even when considered in terms of income. Unfortunately, these results show that this is simply not true. The dynamic strategy outperformed the aggressive strategy by nearly 10% on a gamma basis and by over £100,000 on an certainty equivalent basis. Very few households would deem this difference negligible. Indeed, they also show just how detrimental strategies that are traditionally thought of as ‘conservative’ can be. This brings to mind the phrase ‘reckless conservatism’, coined by Paul Myners (2001) in his famous report. In the same vein it has been said that these strategies are actually ‘highest risk’ due to the certainty with which they fail to deliver on individuals’ objectives.
In our analysis thus far, we have looked at the total value add available by using a more sophisticated solution to the lifetime consumption problem. We next decompose the value add more granularly to better understand the incremental value available from each component of the full adaptive dynamic solution.

**Utility optimal static asset allocation versus buy and hold**

The portfolios that are the best in the sense of the expected utility objective amongst all static MVO portfolios on the efficient frontier have been calculated. By limiting ourselves to only considering MVO portfolios, we unfortunately do not necessarily capture the full value add of switching to a utility objective for a fixed portfolio. This was for purely practical reasons as relaxing the MVO constraint would require unjustifiably large adaptations to the iALM system. For these experiments the tolerable portfolio annual loss constraint of 15% was introduced.\textsuperscript{16} The conservative strategy became unviable and was thus rejected.

The results are summarized in Tables 11 and 12. The optimal static portfolio for the retired profile turned out to be quite close to the aggressive MVO profile. The optimal static portfolio for the young profile is slightly less aggressive than the corresponding aggressive MVO portfolio, showing that the “more risk” mantra does not always hold true, even when considering very long investment horizons. The optimal MVO portfolio is still significantly less efficient than the dynamic strategy, showing the limitations of MVO.

<table>
<thead>
<tr>
<th>Dynamic</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1977k</td>
<td>667k</td>
</tr>
<tr>
<td>Optimal static</td>
<td>984k</td>
<td>288k</td>
</tr>
<tr>
<td>Aggressive</td>
<td>890k</td>
<td>288k</td>
</tr>
<tr>
<td>Moderate</td>
<td>349k</td>
<td>-102k</td>
</tr>
</tbody>
</table>

Table 11. Objective function values for alternative profiles

<table>
<thead>
<tr>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td>Govt bonds</td>
<td>2%</td>
</tr>
<tr>
<td>Alternatives</td>
<td>35%</td>
</tr>
<tr>
<td>International equity</td>
<td>63%</td>
</tr>
</tbody>
</table>

Table 12. Optimal static portfolios

\textsuperscript{16} In previous extensive studies of optimal iALM portfolio allocations, these were found to be very close to the MVO efficient frontier provided a tolerable annual portfolio loss parameter was introduced and set to this level. This can be seen in Figure 2.
**Non-adaptive dynamic strategy versus static buy and hold**

Table 13 shows results for portfolio loss tolerances 15% and 100%, i.e. no portfolio loss penalty. In all cases the scenario path independent non-adaptive dynamic strategy outperforms the static asset allocation strategies, with a single exception. For the young profile the aggressive portfolio with 100% loss tolerance outperforms the 100% loss tolerance non-adaptive dynamic profile. This is because the static asset allocation strategy is still somewhat adaptive. If the market returns in a given scenario from the first year to the next potential major rebalancing point are low, the optimizer may decide to increase total portfolio investment in pounds after this rebalancing point. For the dynamic but non-adaptive strategy the optimizer cannot react to individual scenarios in the same way. Since portfolio increases (in quantity terms) must be constant across scenarios, for scenarios with lower market returns the increases in the rebalancing portfolio value increment in pounds will be lower than for the fixed MVO portfolio expected returns which are higher due to the scenarios with higher market returns.

**Table 13. Portfolio loss tolerance (risk tolerance) effects on the objectives of different strategies**

<table>
<thead>
<tr>
<th>Portfolio Loss Tolerance</th>
<th>Young Obj</th>
<th>Old Obj</th>
<th>Portfolio Loss Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive 100%</td>
<td>1,302,173</td>
<td>344,773</td>
<td>100%</td>
</tr>
<tr>
<td>Aggressive 15%</td>
<td>890,157</td>
<td>288,441</td>
<td>15%</td>
</tr>
<tr>
<td>Dynamic 100%</td>
<td>1,967,466</td>
<td>671,329</td>
<td>100%</td>
</tr>
<tr>
<td>Dynamic 15%</td>
<td>1,976,537</td>
<td>667,167</td>
<td>15%</td>
</tr>
<tr>
<td>Moderate 100%</td>
<td>348,687</td>
<td>-102,070</td>
<td>100%</td>
</tr>
<tr>
<td>Moderate 15%</td>
<td>348,686</td>
<td>-102,070</td>
<td>15%</td>
</tr>
<tr>
<td>Non-adaptive Dynamic 100%</td>
<td>1,212,499</td>
<td>401,175</td>
<td>100%</td>
</tr>
<tr>
<td>Non-adaptive Dynamic 15%</td>
<td>938,084</td>
<td>307,261</td>
<td>15%</td>
</tr>
</tbody>
</table>

Figure 4. Asset allocations of the individual optimal static portfolios
The lack of sensitivity to the portfolio loss tolerance of the iALM dynamic strategy suggests, at least for the profiles tested here, that loosely limiting capital loss is not all that costly in terms of expected utility. We would argue that ordinary investors place a non-trivial value on these @soft@ capital guarantees which are achieved virtually cost free.

When considering goals based investing, most practitioners only think of matched liability driven investment (LDI) which usually leads to a massive increase in volatility of absolute, rather than liability relative, returns. This absolute volatility arises due to the long duration of matched LDI assets. In such narrow interpretations, it follows that income and return objectives are then opposing, so that one or the other must be chosen and they can never be accommodated simultaneously.

Investors considering only fixed strategies face the same problem for a different reason. With the highest risk tolerances, both income and return targets require aggressive portfolio strategies which are dominated by equity exposure so that capital volatility and portfolio loss risk are part and parcel of the strategy.

The dynamic iALM strategy manages to achieve on average lower exposures to risky assets for a higher objective/ lifetime utility through dynamic management of all cash flows. For example, it can choose to move someone into lower risk assets once this is affordable relative to their goal target. This will depend mainly on the average time horizon left and the steepness of the gradient of the goal utility between their acceptable and desirable spending levels. This flexibility also allows the dynamic strategy to start less aggressively.

The 15% portfolio loss tolerance used here is possibly quite large. For lower portfolio loss tolerances the limitation imposed becomes more active. At zero loss tolerance all wealth will be held in cash by the system. However, with the fully dynamic strategy there can be a soft cap to portfolio losses at no significant cost to expected lifetime spend.

Figure 5 shows graphically the objective function differences in percentages between the non-adaptive dynamic and static aggressive MVO strategies from the figures in Table 13 for 15% portfolio loss tolerance.
Figure 5. Relative change of objective from static aggressive to path Independent non adaptive dynamic (life-stage) strategies

**Initial asset allocations for all portfolio strategies**

The initial asset allocations of different strategies are shown in Table 14 and Figure 6. These are the implementable decisions for advisor recommendations and individual implementation which are hedged in the full /ALM dynamic strategy against all future scenarios generated by the system.

<table>
<thead>
<tr>
<th></th>
<th>Dynamic Young</th>
<th>Dynamic Old</th>
<th>Dynamic_non-Adaptive Young</th>
<th>Dynamic_non-Adaptive Old</th>
<th>Conservative</th>
<th>Moderate</th>
<th>Aggressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>dmeq</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>inteq</td>
<td>100.0%</td>
<td>82.9%</td>
<td>100.0%</td>
<td>75.7%</td>
<td>0.0%</td>
<td>22.4%</td>
<td>80.7%</td>
</tr>
<tr>
<td>longs</td>
<td>0.0%</td>
<td>13.2%</td>
<td>0.0%</td>
<td>23.4%</td>
<td>12.2%</td>
<td>28.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>corps</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>commod</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>property</td>
<td>0.0%</td>
<td>1.3%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>6.0%</td>
<td>71.0%</td>
<td>13.9%</td>
</tr>
<tr>
<td>alts</td>
<td>0.0%</td>
<td>2.6%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>9.5%</td>
<td>13.5%</td>
<td>35.7%</td>
</tr>
<tr>
<td>tcash</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>3.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>cash</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 14. Initial asset allocations

Note that for the young profile the starting equity allocations of the dynamic and the non-adaptive dynamic strategies are both fully invested in international equities, but for the retired profile the dynamic strategy has a larger equity proportion than the non-adaptive dynamic strategy. In general, the fundedness of each individual will have a significant impact on whether risk control or upside is the more important, as it will determine which region of the goal utility will be attained.

Figures 7 to 11 show the prospective future portfolio evolutions corresponding to the initial portfolio allocations of Figure 6. Retirement dates are shown by the vertical red lines and the vertical black lines denote possible major portfolio rebalance points not necessarily used by the optimizer for the single spending goal profiles considered here.
Figure 7. Dynamic young profile expected asset allocation

Figure 8. Dynamic young profile with low loss tolerance expected asset allocation

Figure 9. Non-adaptive dynamic young profile expected asset allocation
The overall shape and quantities of the prospective allocations to assets differ quite significantly. For example, there is a far larger allocation to long bonds in the approximating non-adaptive dynamic strategies. This is likely because the dynamic strategy has far more de-risking/hedging power and need not rely only on boosted exposure to matching assets. Of course matching assets have risk management potential; but the fully dynamic strategy can throttle back exposure to risk as projected outcomes deteriorate in terms of utility. In a non-dynamic setting the performance drag of hedging assets must be carried in a portfolio through both good and bad times. On the other hand, when increasingly large dynamic interventions are possible to respond to progressively more severe penalties the situation is greatly improved. It is this dynamism, coupled with the concavity of the utility objective, that jointly create the significant value unlocked by dynamic adaptive strategies. This explanation also shows that the resulting gains are not merely due to “over-fitting” or “fitting to model”. Figures 7, 8 and 10 reveal that the dynamic strategy uses both flexibility and matching, but with a reduced need for reliance on lower return assets.
Models are simplified representations of reality, capturing the most critical components of the problem. Simply put, non-dynamic strategies are not realistic representations of how people actually approach the lifetime consumption problem. They ignore the interventions that investors will make.

We have heard the argument made in the industry as modelling exercises are redone at regular intervals. Revisiting the problem at regular intervals using a non-adaptive model is fundamentally different from optimal decision making that accounts for future adjustability. It is not a simplified representation of the adaptive problem.

It is interesting to note in Figure 9 that the prospective expected allocations for the young non-adaptive dynamic strategy looks similar to the heuristic rule of gradually decreasing the share of equity and increasing the share of bonds in the portfolio over an individual’s lifetime. The pattern is significantly less prominent in Figure 7 for the adaptive dynamic model. Thus the non-adaptive dynamic framework generates a life-staging approach to portfolio evolution over a lifetime which can be seen to be less effective here than the fully dynamic approach. The classical thinking for the life-staging approach is that as an individual’s outstanding present value salary asset decreases in a bond-like manner, it drives the need for bond investments to go up.

### 5.2 Target spending achievement

In the following tables the target (acceptable) spending level is taken to be £40k per annum in all cases.

<table>
<thead>
<tr>
<th></th>
<th>Profile A pre-ret</th>
<th>Profile A post-ret</th>
<th>Profile B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>45%</td>
<td>80%</td>
<td>70%</td>
</tr>
<tr>
<td>Fixed allocation _Conservative</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Fixed allocation _Moderate</td>
<td>1%</td>
<td>25%</td>
<td>10%</td>
</tr>
<tr>
<td>Fixed allocation _Aggressive</td>
<td>15%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>0%</td>
<td>45%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Table 15. Probabilities of achieving the acceptable target spending level**

<table>
<thead>
<tr>
<th></th>
<th>Profile A pre-ret</th>
<th>Profile A post-ret</th>
<th>Profile B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>39k</td>
<td>46k</td>
<td>39k</td>
</tr>
<tr>
<td>Fixed allocation _Conservative</td>
<td>37k</td>
<td>10k</td>
<td>22k</td>
</tr>
<tr>
<td>Fixed allocation _Moderate</td>
<td>36k</td>
<td>22k</td>
<td>29k</td>
</tr>
<tr>
<td>Fixed allocation _Aggressive</td>
<td>37k</td>
<td>35k</td>
<td>36k</td>
</tr>
<tr>
<td>Fixed spending</td>
<td>39k</td>
<td>37k</td>
<td>29k</td>
</tr>
</tbody>
</table>

**Table 16. 30% spending levels**
Table 15 emphasizes just how inefficient the lower risk strategies are. (See the appendix for more details.) The success probabilities for the moderate strategy are vastly worse than those of the aggressive or dynamic strategies. The same holds true for the aggressive strategy relative to the dynamic strategy. Similar remarks apply to the 30% spending levels shown in Table 16 post retirement, which for the dynamic strategy virtually correspond to or surpass the £40k target spending level.\(^\text{17}\) In the authors’ experience, the 30% spending levels (or similar) are used by some advisors to provide drawdown limits beyond which a client should not venture. Looked at another way, 30% is thus regarded as a tolerable risk of ruin with a standard portfolio allocation strategy.

We hope that the success probabilities shown in Table 15 represent a powerful and tangible demonstration for readers of how much difference advanced planning techniques can make. Many modelling practitioners in industry realise that the fully dynamic approach to financial planning is an improvement on current practice and actually implementable today, but they often seem to jump to the conclusion that its value add will be very marginal. These results show the value added by fully dynamic strategies indisputably, as there is no rational debate to be had concerning a change of 15% to 45% of target achievement not being significant.

6. Conclusion

In this paper we have used iALM to demonstrate the effects of using a dynamic goal-based holistic stochastic approach to solving the life-cycle consumption problem. We evaluate and decompose the relative value-added for individual clients or pension fund members by using these technologies to replace the current bases of advice given by industry to clients. These include Markowitz mean variance optimized portfolios with varying degrees of risk aversion; specific goal funds, for example, to cover an individual household’s future school or university fees; life-stage funds; and fixed real post-retirement spending, for example, by means of fixed defined contribution pension fund withdrawals, or purchase of an indexed fixed annuity at retirement.\(^\text{18}\) The method used was to embed each of these industry standard bases in the dynamic stochastic planning system and then evaluate their relative effectiveness in meeting an individual’s goals by means of two statistics of the optimal expected utility of lifetime consumption of the system. The two intuitive comparative statistics used were gamma (Blanchett and Kaplan, 2013), based on certainty equivalent consumption, and the certainty equivalent gap, representing the additional initial capital required to give the inferior strategy the same expected utility as the superior one. These simple statistics were supplemented by detailed comparisons of initial portfolio allocations and prospective future portfolio evolutions, spending target achievement probabilities and the extensive use of graphs and tables. The results are surprising, even to us, as the advantages of dynamic flexibility embodied in the holistic iALM model can be seen to significantly outperform the other approaches -- fixed post-retirement spending in real terms, as with indexed fixed annuities, being particularly bad.

Since we tested the detrimental effects of fixed spending and fixed portfolio strategies separately, it would be interesting to see the detrimental effects of both of these limitations together. We suspect that the result might be even worse than sum of their individual effects, but we leave verification of this to future research.

In summary, we hope that the results of this paper will go some way to convincing the pensions and financial advisory industries that the holistic dynamic stochastic strategies required to address members and clients’ actual needs (MacDonald et al., 2013; US GAO, 2014; Laster, 2014) are worth the extra effort.

\(^\text{17}\)Recall that pre-retirement fixed spending for the aggressive MVO strategy was set at the dynamic optimal per annum average.

\(^\text{18}\) In earlier work we have experimented with the benefits and optimal timing of annuity purchase to find that these depend critically on the characteristics of the actual products offered to individuals by the industry.
Acknowledgement

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-13) under grant agreement no 289032 (HPC Finance).

References


Appendix

A.1 Profile A: 30 year old single with no savings

A1. Optimal dynamic asset allocation and spending through life

![Diagram of terminal wealth](image1)

*Desired value:* £0  **Acceptable value:* £0  ***Expected value:* £275,478

Client-Specified Range  Distribution Across All Scenarios  Likelihood of Achieving Level

![Diagram of living expenses](image2)

*Expected value:* £30,465  **Acceptable value:* £40,000  ***Desired value:* £50,000

Client-Specified Range  Distribution Across All Scenarios  Likelihood of Achieving Level

![Diagram of retirement spending](image3)

*Acceptable value:* £40,000  **Expected value:* £52,009  ***Desired value:* £70,000

Client-Specified Range  Distribution Across All Scenarios  Likelihood of Achieving Level

Objective value = 1,997,466
A2. Static conservative fixed asset allocation

Objective= -886,506
A3. Static moderate fixed asset allocation

Objective= 348,687
A4. Static aggressive fixed asset allocation

Objective= 1,302,173
A5. Optimal dynamic asset allocation with partly fixed spending

The spending is fixed to the expected amount of spending of the fully dynamic solution (A1) pre-retirement, after retirement - spending and asset allocation is optimally dynamic.

Objective=1,771,880
A.2 Profile B: 65 year old retired single with £600k savings

B1. Optimal dynamic asset allocation and spending

![Diagram of Terminal Wealth](image)

*Desired value: £0  **Acceptable value: £0  ***Expected value: £224,760

Client-Specified Range | Distribution Across All Scenarios | Likelihood of Achieving Level

![Diagram of Retirement Spending](image)

*Acceptable value: £40,000  **Expected value: £45,388  ***Desired value: £70,000

Client-Specified Range | Distribution Across All Scenarios | Likelihood of Achieving Level

Objective: 671,329
B2. Static conservative fixed asset allocation

Objective: -865,730
B3. Static moderate fixed asset allocation

Objective: -102,070
B4. Static aggressive fixed asset allocation

Objective: 344,773
B5. Optimal dynamic asset allocation with fixed spending

Objective: -298,678