Vol Target Options

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February 7, 2014

Outline

1. Introduction
2. Pricing Effects
   - Discretisation effect
   - Skew
   - Stochastic rates
3. Greeks
   - Vega
   - Theta
   - Delta & Gamma
   - Hedge P& L
   - Jump sensitivity

The Basic Idea

- Basket split between risky asset and cash
- Chose weight of risky asset w to keep volatility of the basket constant.

$$w = \frac{\sigma^{target}}{\sigma^{hist}}$$

- Systematic differences between current and historical volatility prevent the basket behaving as true constant vol asset.

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Simple product

$$I_{n+1} = I_n + \frac{w_n I_n}{S_n} \left(S_{n+1} - S_n \right)$$

$$w_n = \frac{\sigma^{target}}{\sqrt{\frac{252}{m} \sum_{i=0}^{m-1} \log^2 \left(\frac{S_{n-i}}{S_{n-i-1}} \right)}}$$

Motivation

The value of an option on a vol target is roughly the Black Scholes value with $\sigma^{eff} \approx \sigma^{target}$

$$V^{BS}(I, \sigma^{eff}, K, T)$$

Why derive closed form approximations to greeks and price?

- Improve Intuition.
- Hedge simulations

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Discretisation effect - Intuition

- Even in Black Scholes model σ^{hist} is random.

$$\mathbb{E}[\sigma^{hist^2}] = \sigma^{BS^2}$$

- Convexity raises the effective volatility

$$\mathbb{E}\left[\frac{1}{\sigma^{hist^2}}\right] = \left(\frac{m}{m-2}\right)\frac{1}{\sigma^{BS^2}}$$

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Discretisation effect - Approximate result

- Business day volatility

$$\sigma^{target} \to \sigma^{target}\sqrt{\frac{m}{m-2}}$$

- Calendar day vol. Proportion of variance that falls over weekend $= \alpha$

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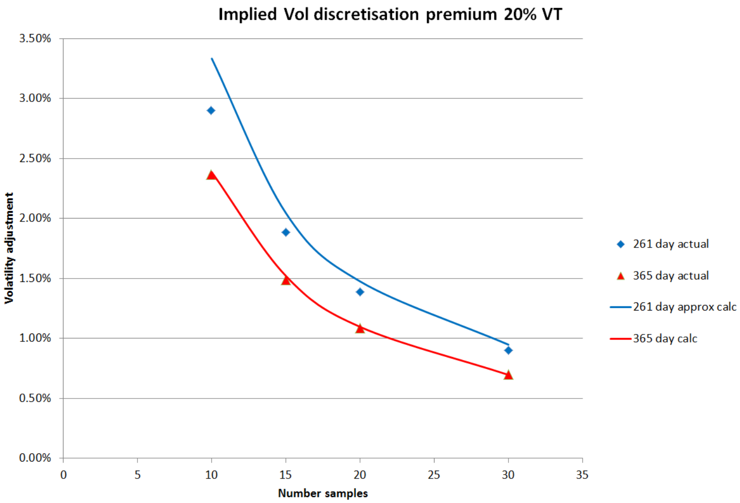
Discretisation effect - Demonstration



Discretisation effect - Removal

Use a continuous time version of historical vol, over time period Δ, to derive formulae without discretisation effect.

$$\sigma_t^{hist} = \sqrt{\frac{1}{\Delta}\int_{t-\Delta}^{t} \sigma_u^2 du}$$

Historical volatility averaging period

$$\Delta \approx \frac{m}{252}$$

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Skew - Intuition

- As underlying S drifts down its volatility tends to increase.
- Historical volatility takes time to catch up.
- Risky weight w too high and σ^{eff} of the index is larger than σ^{target} for paths where the index drops.

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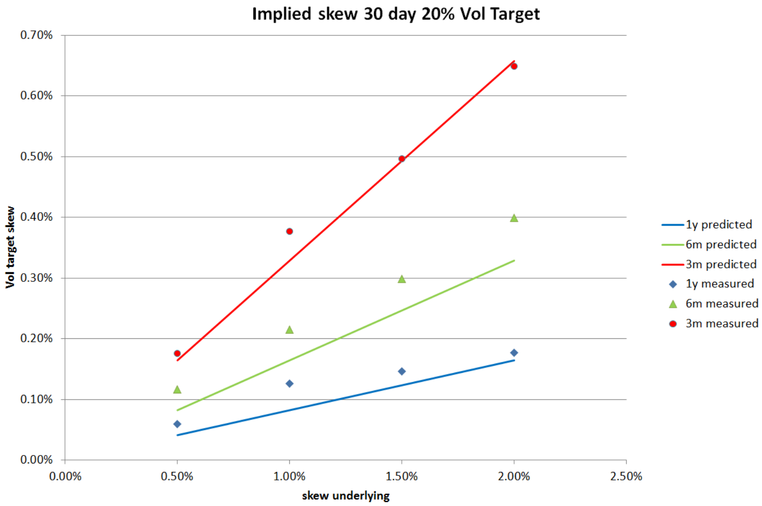
Skew - Approximate result

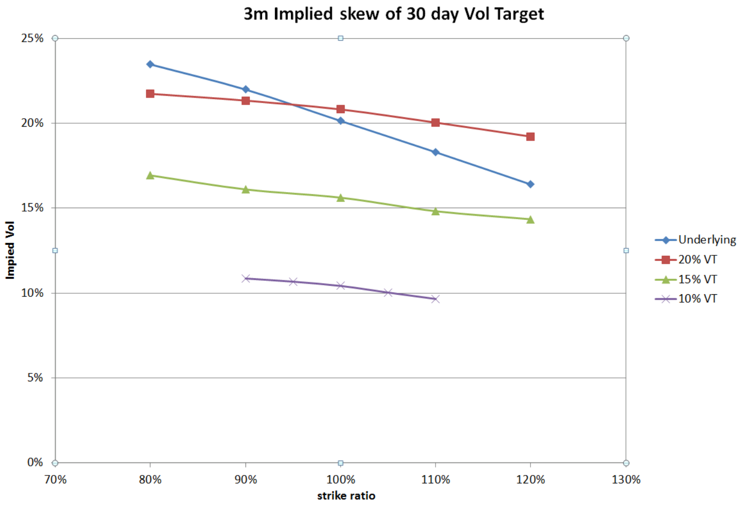
Define skew to be the change in implied volatility for a 10% move in underlying. Assume constant skew in implied vol surface of S.

$$Skew \to Skew\frac{\Delta}{T}$$

Skew - Demonstration.



Skew - Demonstration.



Skew simulations - Morrison, Tadrowski

Historical volatility from exponentially weighted mean.
Equivalent to 0.8 year unweighted mean.

- Calibrate jump model to 1.5% skew on sx5e at 2 years.
- Effective Vol skew around 0.53%
- ATM vol bump 1.65%

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Calibrate jump model to skew on S&P500 at 3 years.

- σ^{eff} skew under local vol $\approx$ σ^{eff} skew under local vol plus jumps.
- ATM vol bump $\approx$ 1.8%

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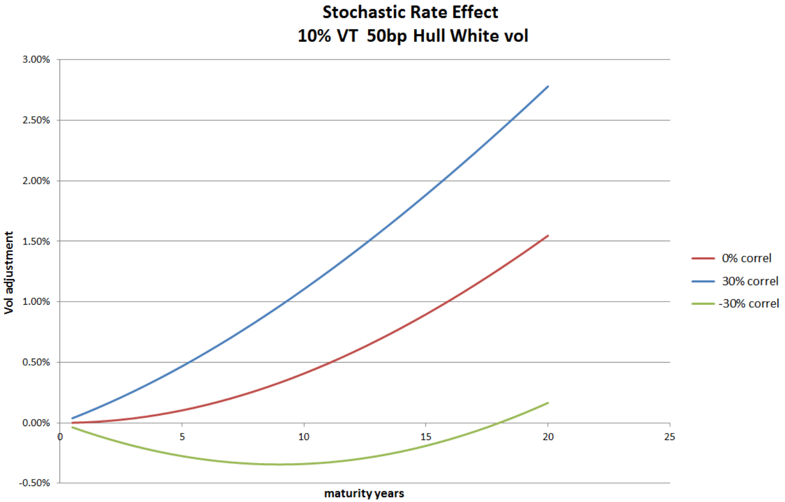
Stochastic Rates - Approximate result

Risky and riskless assets grow at short rate following a Hull White process with normal volatility σ^{short}. Negligible speed of mean reversion.

$$\sigma^{eff} \approx \sigma^{target}\sqrt{1 + \rho\frac{\sigma^{short}}{\sigma^{target}}T + \left(\frac{\sigma^{short}}{\sigma^{target}}\right)^2 \frac{T^2}{3}}$$

Stochastic Rates - Approximate result



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Vega - Intuition

Volatility of underlying rises from σ to $\sigma + \delta\sigma$.

$$V^{BS}(I, \sigma^{eff}(\delta\sigma), K, T - \epsilon)$$

- While historical volatility is too low the leverage is to high and the Index evolves with σ^{eff} higher than σ^{target}

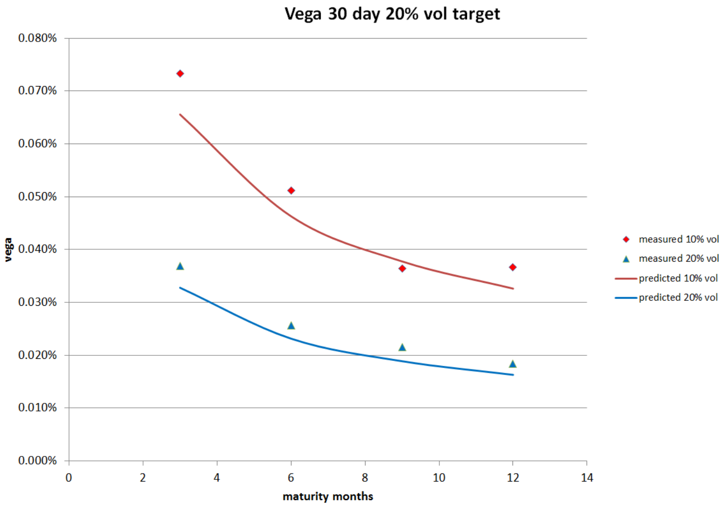
Vega - Approximate result

For an option maturing after T years.

$$\sigma^{eff} \approx \sigma^{target} \sqrt{1 + \frac{\delta\sigma}{\sigma} \frac{\Delta}{T}}$$

$$Vega^{BS} \rightarrow Vega^{BS} \frac{\sigma^{target}}{\sigma} \frac{\Delta}{2T}$$

Vega - Demonstration.



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Theta - Intuition

Underlying remains constant for period ϵ.

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- Time decay of option will decrease its value.
- Period of zero volatility decreases σ^{hist} so σ^{eff} increases.

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Theta - Approximate result

$$\sigma^{eff}(\epsilon) = \sigma^{target}\sqrt{1 - \frac{\Delta}{T-\epsilon}\log\left(1-\frac{\epsilon}{\Delta}\right)}$$

$$\frac{\partial \sigma^{eff}}{\partial \epsilon} = \frac{\sigma^{target}}{2T}$$

$$\theta^{BS} \rightarrow \theta^{BS} + Vega^{BS}\frac{\partial \sigma^{eff}}{\partial \epsilon}$$

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Delta - Intuition

Underlying moves from level at last fixing S_0 to S

$$V^{BS}(I(S), \sigma^{eff}(S), K, T)$$

- The value of I changes in proportion to current weight.
- σ^{eff} changes through addition of $\log^2\left(\frac{S}{S_0}\right)$ term to $\sigma^{hist^2}\Delta$

Delta - Intuition

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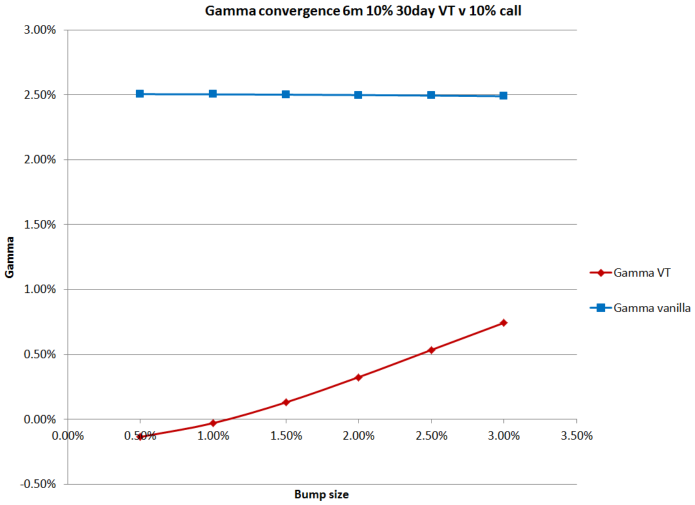
Delta - Approximate result

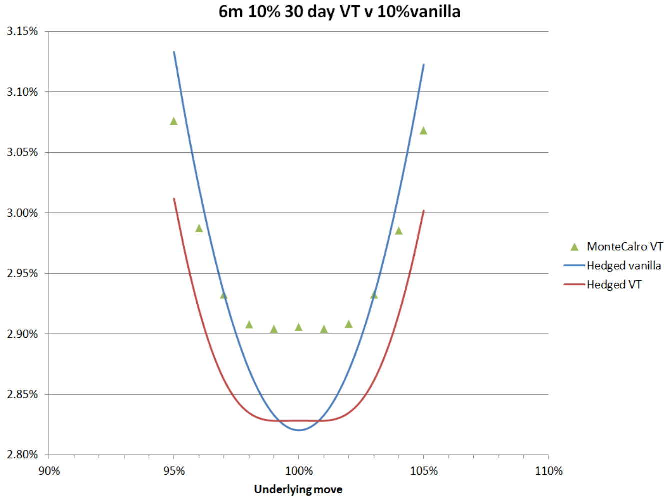
Effective volatility σ^{eff} is even in $\log(S)$.

$$\frac{\partial V}{\partial S} = \frac{\partial V}{\partial I} \frac{\sigma^{target}}{\sigma}$$

Gamma - Result



Gamma - Result



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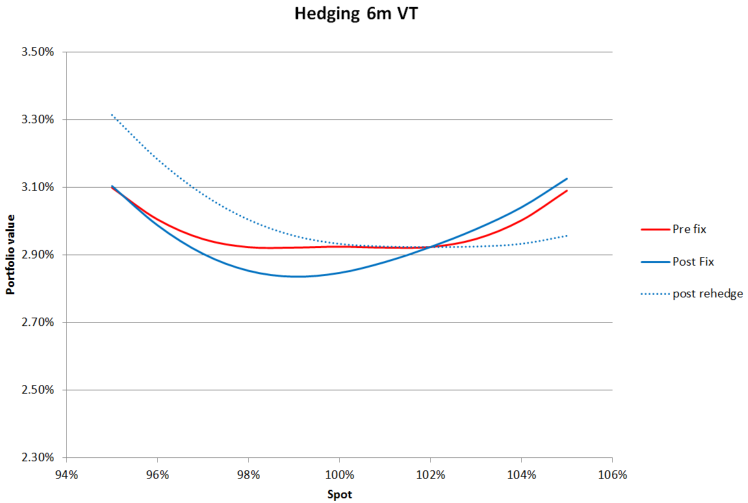
Hedge P & L


Hedging 6m VT
3.50%
3.30%
3.10%
2.90%
2.70%
2.50%
2.30%
Portfolio value
94%
96%
98%
100%
102%
104%
106%
Spot
Pre fix
Post Fix
post rehedge

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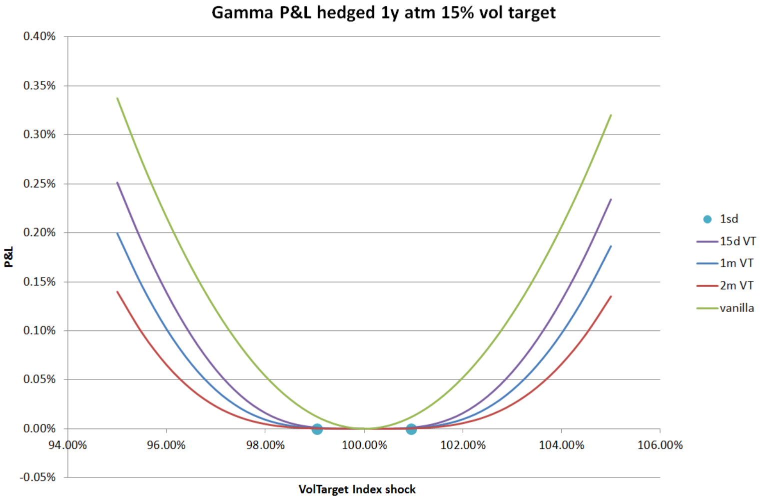
Jump sensitivity - How big is a jump?



Summary - σ^{hist} measurement period

As Δ increases

- Fair Value decreases because less discretisation effect.
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Summary - Largest pricing effects

- Jumps
- Stochastic rates
- Discretisation

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- Jumps
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Some simulation studies I

- Steven Morrison, Laura Tadrowski
  Guarantees and target volatility funds.
  Barrie and Hibbert research paper Sept 2013.
- Maximilian Nelte, Peter Roche
  Controlling volaility to reduce uncertainty.
  Risk Sept 2010.