Computational Challenges of Stochastic Modelling in Life Insurance

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Computational Challenges of Stochastic Modelling in Life Insurance: Outline of presentation

» Background to the problem
» A potential solution: Proxy modelling and Least Squares Monte Carlo
» Example analysis: Multi-year projection
» Summary and Q&A
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Background to the problem
The basic computational challenge

Life insurance companies have to value liabilities containing complex embedded options

Example policy:

» Annual return credited to policyholder is calculated as
  » Credited return (t) = max (Fund return (t) – 1.5%, 2%)
  » i.e. Policy Account (t) = Policy Account (t-1) * (1 + Credited return (t))
  » Policyholder exits the policy at year 10
  » Asset fund invested in diversified portfolio of investment-grade corporate bonds
  » Typical portfolio may contain millions of such policies, each with different characteristics

Liability payoff is complex

» Depends on multiple risk factors
» Path dependent
» Requires Monte Carlo valuation
» Cash flow rules implemented within ‘ALM models’, often inefficiently

Typically ~10,000 scenarios; Could take several hours

Typically annual or monthly time-steps over 30+ years
It gets worse...

Monte Carlo valuation needs to be repeated in many (future) scenarios e.g.:

Regulatory or economic capital requirements often defined as a 1-year Value-at-Risk

Own Risk and Solvency Assessment (ORSA) requires the projection of capital over multi-year paths
Potential solutions

» Bigger, faster, technology

» Efficient model design
  – Portfolio compression

» Variance reduction techniques

» Proxy methods
  – Replicating portfolios
  – Curve fitting
  – Least Squares Monte Carlo (LSMC)
A potential solution: Proxy modeling and Least Squares Monte Carlo
Proxy Functions

Value is represented by a simple function. Typically these functions are polynomials with ~50 individual terms. Exact number will depend on the number of risk factors and the complexity of the portfolio.

Liability Value = $a + b.RF_1 + c.RF_2 + ... + k.(RF_1)^2.RF_3 + ....$

\(a, b, c\ldots\) are constants. \(RF_x\) represents risk factors
Proxy Functions
Curve Fitting

The curve fitting approach fits a polynomial function (i.e. multi-dimensional surface) through a set of chosen points with associated ‘accurate’ valuations (using ~5,000 market consistent scenarios).

This is not an efficient use of the overall scenario budget. The ALM model is still a constraint and an insurer may only be able to fit through a few hundred points using this approach.

### Scenario Budget

<table>
<thead>
<tr>
<th>Scenario Type</th>
<th>Number of Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year VaR Scenarios</td>
<td>50</td>
</tr>
<tr>
<td>Market Consistent Scenarios</td>
<td>5,000</td>
</tr>
<tr>
<td>Total Scenarios</td>
<td>250,000</td>
</tr>
</tbody>
</table>

However, limited set of fitting points may result in poor fit for certain areas of the curve.

Accurate value for a selection of points on curve – requires interpolation.

Function fitting
Proxy Functions
LSMC

The LSMC approach increases the number of fitting points but gives up the accuracy of the associated valuations due to the reduced number of market consistent scenarios.

The clever bit is that as long as the market consistent scenarios are independent the fit is much better than curve fitting for the same scenario budget.

<table>
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<tbody>
<tr>
<td>1 year VaR Scenarios</td>
<td>50,000</td>
</tr>
<tr>
<td>Market Consistent Scenarios</td>
<td>2</td>
</tr>
<tr>
<td>Total Scenarios</td>
<td>100,000</td>
</tr>
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Inaccurate value for every point on curve – however least squares regression captures the overall shape of the curve.
Curve fitting and LSMC are two extremes in a spectrum of outer/inner scenario allocations.
Suppose we have a fixed number of scenarios that can be passed through the ALM model

Scenario budget = # outer scenarios x # inner scenarios?
   - How to optimally choose # outers and # inners?

Some evidence that it is optimal to choose small number of inners and large number of outers

Source: Cathcart, M. (2012), Monte Carlo simulation approaches to the valuation and risk management of unit-linked insurance products with guarantees, Heriot-Watt University PhD thesis.
Full nested stochastic – value every point, so reduces expert judgement and very **accurate** but requires impractical scenario budget.

**Curve fitting** – value a selection of points, so requires significant expert judgement since must choose points. **Accurate** on chosen points but not between points where interpolation is required. Manageable scenario budget.

**LSMC** – value every point, so reduces expert judgement. Not accurate for individual points but regression insures **accuracy**. Optimal allocation of scenario budget.

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* Of function across full range of stresses to risk drivers

* To configure the fitting process, reduction of upfront knowledge of liability behaviour desirable

* Outer stress scenarios X number MC scenarios required
Process for Proxy Generation using LSMC

» Four different steps are necessary to determine the proxy function

1. Identify risk factors and generate fitting points
2. Calculation of Asset/Liability PVs at each point
3. Optimisation to fit proxy function
4. Validation of proxy function against accurate valuations
Choosing fitting points

» Note that stresses don’t need to be RW – the aim is to span the space in an efficient way

» Some evidence that uniform sampling is superior to real-world sampling (for the purpose of estimating 1-year VaR)
   – Commonly via quasi-random sampling (e.g. Sobol) for high dimensional problems

Source: Cathcart, M. (2012), Monte Carlo simulation approaches to the valuation and risk management of unit-linked insurance products with guarantees, Heriot-Watt University PhD thesis.
Selecting and calibrating the proxy function

» Decide on a set of candidate terms for the (polynomial) function
  – Maximum order
  – Maximum order of cross terms

» Use step-wise regression algorithm to efficiently search space of candidate models

» At each step, assess goodness of fit using metric such as Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC)
Proxy Functions Validation

The final step is to validate the polynomial function. In this case a selection of points are chosen and full valuations are performed in the ALM model and compared against the values from the polynomial function.

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Example analysis: Multi-year projection
LSMC: From One Year to Multiple Years

One year LSMC

Multi-year LSMC
Example product and proxy function

Consider example policy introduced earlier:

- Annual return credited to policyholder is calculated as
  - Credited return (t) = max (Fund return (t) – 1.5%, 2%)
  - i.e. Policy Account (t) = Policy Account (t-1) * (1 +Credited return (t))
  - Policyholder exits the policy at year 10

- Asset fund invested in diversified portfolio of investment-grade corporate bonds

Fit polynomial proxy function for market-consistent value:

- MC Value = f(yield curve level, yield curve slope, 10-year BBB spread, policy account, yield curve volatility, time)

- Fitted polynomial f() has 32 terms
Case study: Validation Results at years 1, 5 and 9
Multi-Year Stochastic Projections: Liability Value – Asset Portfolio Value

Note: 10,000 ‘real-world’ scenarios at annual time-step over 10 years

= 100,000 market-consistent valuations!
Multi-Year Stochastic Projections: Liability Value – Asset Portfolio Value in Year-5

- Strong exposure to credit spread behaviour

- High risk-free rates also bad for shareholder in this product
  - But not so significant as high rates also reduce liability value
Reverse Stress Test:
Economic scenario path for worst ranking scenario

- Rank year-5 real-world scenarios by market-consistent deficit
- This scenario has the biggest deficit at year-5
- The economic scenario is consistent with previous scatterplots
- M-C deficit at year-5 is 0.60 (99th percentile in earlier chart was 0.30)
Summary
Computational Challenges of Stochastic Modelling in Life Insurance: Summary

» Life insurance companies have to value liabilities containing complex embedded options

» Requires (slow) Monte Carlo valuation

» Requirement to *calculate capital* requires projection of value

» Leads to nested simulation problem

» Requirement to *project capital* leads to doubly nested stochastic problem

» Growing use of proxy methods to replace computationally intensive valuations

» Polynomials emerging as most popular form of proxy function

» Least Squares Monte Carlo is an efficient method for calibrating such proxy functions