A Practical Robust Long Term Yield Curve Model

M A H Dempster

Centre for Financial Research, Statistical Laboratory
University of Cambridge
& Cambridge Systems Associates Limited

mahd2@cam.ac.uk  www.cfr.statslab.cam.ac.uk

Co-workers: Elena Medova, Igor Osmolovskiy & Philipp Ustinov
Outline

- Introduction
- Multifactor yield curve models
- Difficulties with Gaussian affine models
- Black correction for negative rates
- HPC approaches to calibrating Black models
- Unscented Kalman filter implementation
- Conclusions
Introduction

- In all the world’s major economies low interest rates have prevailed since the 2007-2008 financial crisis which were presaged by more than a decade in Japan.

- This has posed a problem for the widespread use of diffusion based yield curve models for derivative and other structured product pricing and for forward rate simulation for systematic investment and asset liability management.

- Sufficiently accurate for pricing and discounting in relatively high rate environments Gaussian models tend to produce an unacceptable proportion of negative forward rates at all maturities with Monte Carlo scenario simulation from initial conditions in low rate economies.

- The implications for this question of negative nominal rates in deflationary regimes and the currently fashionable multi-curve models remain to be seen.
Variety of Approaches to Yield Curve Modelling

- **Investment bank pricing and hedging** of fixed income products
  - Short term current market data calibration
  - Updated for re-hedging
  - Evaluated by realized hedging P&L

- **Central bank forecasts** for monetary policy making
  - Long term historical estimation for medium term forecasting
  - Updated for next forecast
  - Mainly evaluated by in-sample fit to historical data

- **Consultants and fund managers advice** for product pricing, investment advice and asset liability management over long horizons
  - Long term historical calibration to market data often using filtering techniques
  - Updated for decision points
  - Evaluated by consistency with out-of-sample market data: e.g. prices, returns
High Performance Computing Requirement

- Beginning with work in the Bank of Japan in the early 2000s there is currently considerable research in universities, central banks and financial services firms to develop yield curve models whose simulation produces nonnegative rate scenarios.

- All this work is based on a suggestion of Fisher Black (1995) published posthumously to apply a call option payoff with zero strike to the model instantaneous short rate which leads to a piecewise nonlinearity in standard Gaussian affine yield curve model formulae for zero coupon (discount) bond prices and the corresponding yields and precludes their explicit closed form solution.

- As a result most of the published solutions to Black-corrected yield curve models are approximations and even these require high performance computing techniques for numerical solution but we shall study here an obvious approximation which works extremely well as we shall see and is amenable to cloud computing for speed up.
Multifactor Yield Curve Models
Yield Curve Model Applications

- **Scenario simulation** for predominantly long term asset liability management (ALM) problems in multiple currencies

- **Valuation** of complex structured derivatives and other products and portfolios with embedded derivatives in multiple currencies

- **Risk assessment** of portfolios and structured products
Model Requirements

- Continuous time
- Mean reversion
- Dynamic evolution under both pricing (risk neutral) and market (real world) measures
- Wide range of yield curve shapes and dynamics reproduced (LIBOR)
- Realistic zero lower bound (ZLB) modelling
- Feasible and efficient discount bond price or yield calculation
- Parameter estimation by efficient model calibration to market data to multiple yield curves and currency exchange rates
- Parsimony in parameter specification
- Time homogeneity

Multi-factor Yield Curve Models

- Three broad overlapping classes
  - Short rate models
  - Heath-Jarrow-Morton models
  - Market models
- Most rate variability captured by 3 stochastic factors
  Litterman & Scheinkman (1991)
- The 2 factor affine or quadratic short rate models are insufficient to reproduce the correlation structure of market rate changes but 3 to 5 factors suffice
- The Nelson-Siegel (1987) 3-factor short rate model widely used by central banks has time inhomogeneous parameters and is neither parsimonious nor arbitrage free
- The Diebold-Rudebusch (2011, 2013) version of this model corrects both these faults
  Rebonato (2015)
3 Factor Affine Short Rate Models

- The 3 factors under the pricing (risk-neutral) measure $Q$ satisfy the $A_0(3)$ SDE
  \[ dY_t = \Lambda(\Theta - Y_t)dt + \Sigma \sqrt{S_t} dW_t \]
  \[ [S_t]_{ii} = \alpha_i + \beta_i' Y_t \quad i = 1, 2, 3 \]

- Discount bond prices are given in affine form as
  \[ P_t(\tau) = e^{A(\tau) + B(\tau)' Y_t} \]
  and the instantaneous short rate similarly as
  \[ r_t = \phi_0 + \phi_x' Y_t \]

- Then bond prices and yields are given respectively by
  \[ P_t(\tau) = E^Q \left[ \exp \left( -t^\tau r_s ds \right) \right] \]
  and
  \[ y_t = -\ln P_t(\tau) / \tau \]

- A 3 rate vector satisfies the Ricatti equation
  \[ \frac{\partial R_t(\tau)}{\partial \tau} = \Lambda R_t(\tau) - \frac{1}{2} R_t(\tau) \Sigma \Sigma' R_t(\tau)' + r_t \]

3 Factor Gaussian Extended Vasicek Model

- Specified under P by
  \[ \Lambda := \begin{pmatrix}
  \lambda_{11} & 0 & 0 \\
  \lambda_{21} & \lambda_{22} & 0 \\
  \lambda_{31} & \lambda_{32} & \lambda_{33}
\end{pmatrix} \]
  \[ \Theta := \begin{pmatrix}
  \theta_1 \\
  \theta_2 \\
  \theta_3
\end{pmatrix} \]
  \[ \Sigma := \begin{pmatrix}
  \sigma_1 & 0 & 0 \\
  0 & \sigma_2 & 0 \\
  0 & 0 & \sigma_3
\end{pmatrix} \]
  \[ S := \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix} \]
  \[ r(t) := \delta_0 + \delta_1 y_1(t) + \delta_2 y_2(t) + \delta_3 y_3(t) \]

- This Dai & Singleton \( A_0(3) \) model with 16 parameters is not identified under P unless \( \Theta := 0 \) which is only appropriate to Q and has other difficulties
Joslin-Singleton-Zhu (JSZ) 3 Factor Affine Gaussian Yield Curve Model

- The underlying stochastic differential equation (SDE) for the continuous time evolution of the 3 factors $Y$ is given by

$$dY(t) = \Lambda(\theta - Y(t) + \Pi(t))dt + \Sigma \sqrt{S(t)}dW(t),$$

where now

$$\Pi(t) = k_0 + K_1 Y(t)$$

is the affine state dependent market price of risk vector.

- JSZ estimate the discrete time version with 3 observed yield curve points – rates – fit exactly and extra rates least squares fit approximately as two standard econometric vector autoregression (VAR) models given respectively under the pricing and market (real world) measures $Q$ and $P$.

JSZ (2011)
Economic Factor Model

- A 3 factor extended Vasicek Gaussian model specified under the market measure $P$ by

$$dX(t) = (\mu_X - \lambda_X X(t) + \gamma_X \sigma_X)dt + \sum_{j=1}^{3} \sigma_{1j} dW_j(t)$$

Long rate

$$dY(t) = (\mu_Y - \lambda_Y Y(t) + \gamma_Y \sigma_Y)dt + \sum_{j=1}^{3} \sigma_{2j} dW_j(t)$$

Minus Slope

$$dR(t) = \{k[X(t) + Y(t) - R(t)] + \gamma_R \sigma_R\}dt + \sum_{j=1}^{3} \sigma_{3j} dW_j(t)$$

Unobservable instantaneous short rate

- Its discretization is estimated from CMS swap data with many observed yield curve points – rates – from 1 day (Libor) to 30 years (Treasury) using the EM algorithm which iterates Kalman filtering and maximum likelihood estimation to convergence

- Specifying the constant market prices of risk in terms of volatility units solves the $X$ & $Y$ identification problem and setting them to zero generates the factor pricing process

- This workhorse model has been used for pricing complex products and ALM using daily to quarterly frequency data in US, UK, EU, Swiss and Japanese jurisdictions
State Space Model Formulation

Transition Equation

\[ Y_t = d + \Phi Y_{t-1} + \eta_t, \]

\[ E[Y_t|Y_{t-1}] = d + \Phi Y_{t-1} \]

\[ var(\eta_t) = var(Y_t|Y_{t-1}) = \Omega(Y_{t-1}) := \Omega_t \]

Measurement Equation

Substantial number of observed yields (e.g. 14)

\[ y_t = A + BY_t + \varepsilon_t \]
Calibrating the EFM Model

- Given the vector of parameters $\theta$ this Gaussian extended Vasicek model has rates (zero coupon bond yields) for maturity $\tau := T - t$ of the form

$$y(t,T) = \tau^{-1}[A(\tau, \theta)R_t + B(\tau, \theta)X_t + C(\tau, \theta)Y_t + D(\tau, \theta)]$$

- We interpolate the appropriate swap curve linearly to obtain swap rates at all maturities and then use 1, 3 and 6 month LIBOR rates and the swap curve to recursively back out a zero coupon bond yield curve for each day from the basic swap pricing equation Ron (2000)

- This gives the input data for model calibration to give the parameter estimates $\hat{\theta}$

- Calibration is accomplished using the EM algorithm which iterates successively the Kalman filter (KF) and maximum likelihood estimation from an initial estimate $\theta_0$

- At each iteration multi-extremal likelihood optimization in $\theta$ is accomplished using a global optimization technique followed by an approximate conjugate direction search

- The procedure is run on a Dell 32 Intel core system using parallelization techniques and we have also investigated the use of cloud computing for these calculations
Goodness of Fit to Historical Yield Curves
Mean level of yields over 2003 for historical and simulated weekly data

Weekly standard deviation of yields over 2003 for historical and simulated data

Dempster, Medova & Villaverde (2010)

- Longer term out-of-sample yield curve prediction has recently been independently found to be superior to the arbitrage-free Nelson-Siegel model of Christensen, Diebold & Rudebusch (2011) widely used by central banks
Monte Carlo Structured Deal Valuation

- OTC deal valuation may require several yield curve estimates together with CMS swap rates and cross currency rates which are all assumed correlated with fixed values.
- The estimated factor dynamics of \((X,Y,R)\) are simulated forward under the Q measure for pricing with the fixed market prices of risk set to 0.
- The corresponding curves and FX rates are simulated to maturity together with a daily time step from respectively the valuation day yield curve estimates and FX data using 10,000 paths.
- For OTC client deals optionality is typically in the form of bank cancellation rights (without compensation) at prescribed dates – usually at all reset dates after some initial period from inception.
- We use an augmented version of a sub-optimal cancellation rule due to Andersen (1999) which relies on a score function \(s_t(x,y,r)\) and cancels if \(s_t < s_t^*\).
- The exercise thresholds \(s_t^*\) are determined by a separate set of 10,000 paths for \((X,Y,R)\) as the discounted value of all the remaining net payouts to the bank along the average factor path.
Difficulties With Gaussian Affine Models
JSZ Affine Model Fit Numerical Instability

Dempster, Evans & Medova (2014)
JSZ Model 25 Year Yield Curve Projections

Dempster, Evans & Medova (2014)
EFM Model Euro 10 Year Rate for 30 Years

Quantiles based on 100,000 scenarios

Market data
Black Correction for Negative Rates
Nonlinear 3-Factor Black Model

- In a posthumously published paper Fisher Black (1995) suggested correcting *a priori* a Gaussian short rate model for a shadow short rate $r$ to give the actual short rate as

$$r_{\text{actual},t} := \max[0, r_{\text{shadow},t}] := 0 \vee r_{\text{shadow},t}$$

- Applied to an affine 3-factor Gaussian yield curve model such as that of JSZ or our EFM model this yields a hard nonlinear estimation problem posed by the bond price

Joslin, Singleton & Zhu (2011) \[ P_t(\tau) = E^Q \exp[-\int_t^{t+\tau} 0 \vee r_{\text{shadow},s} \, ds] \]

- Such models have been studied in the 2-factor case by the Bank of Japan and at Stanford but their discount bond pricing (rate) PDE methods do not easily extend to 3 factors.

## 3-Factor Black Model Stylized Properties

<table>
<thead>
<tr>
<th>Stylized Fact Properties</th>
<th>Yield Curve Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CIR</td>
</tr>
<tr>
<td>Mean Reverting Rates</td>
<td>Yes</td>
</tr>
<tr>
<td>Nonnegative Rates</td>
<td>Yes</td>
</tr>
<tr>
<td>Stochastic Rate Volatility</td>
<td>Yes</td>
</tr>
<tr>
<td>Closed Form Bond Prices</td>
<td>Yes</td>
</tr>
<tr>
<td>Replicates All Observed Curves</td>
<td>No</td>
</tr>
<tr>
<td>State Dependent Risk Premia</td>
<td>No</td>
</tr>
<tr>
<td>Good for Long Term Simulations</td>
<td>No</td>
</tr>
<tr>
<td>Slow Mean Reversion Under Q</td>
<td>No</td>
</tr>
<tr>
<td>+ve Rate/Volatility Correlation</td>
<td>No</td>
</tr>
<tr>
<td>Effective in Low Rate Regimes</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 1. Properties of evaluated yield curve models with regard to stylized facts

*Rate volatilities are piecewise constant punctuated by random jumps to 0 at rate 0 boundary hitting points.
Black Model 10 Year Gilt Rate
50 Year Predicted Distribution 2011-2061

Quantiles based on 10,000 scenarios
HPC Approaches to Calibrating Black Models
3 Factor Black Model Approaches

- The differences between current approaches to Black models based on 3 factor affine shadow rate models may be categorized in terms of handling the three steps crucial to the solution process:
  - Method of inferring (3 factor) states from observed market rates
    - inverse mapping or least squares
    - extended or iterated extended Kalman filter (EKF or IEKF) with piecewise linearization
    - unscented Kalman filter (UKF) with averaged multiple displaced KF paths
  - Method of parameter estimation
    - method of moments
    - maximum likelihood (MLE) or quasi maximum likelihood (QMLE)
  - Method of calculating bond prices or yields
    - Monte Carlo simulation
    - PDE solution
    - approximation
Monte Carlo Bond Pricing

- Calibration of the nonlinear Black model with any underlying 3 factor Gaussian shadow rate model is more computationally intensive than for the underlying affine model.

- Dempster, Evans & Medova (2014) use cloud facilities and Monte Carlo simulation with a JSZ 4 yield curve point model.

- In more detail:
  - For short rates the closed form numerical rate calculations of Kim & Singleton (2011) are used.
  - For long rates the averages of Monte Carlo forward simulated paths -- which automatically take account of the convexity adjustment otherwise required for this model -- are used.

- With this approach filtering a multi-curve EFM model for OTC structured derivative valuation becomes very computationally intensive.
PDE Bond Pricing

- A possible key to calibration of both the JSZ and EFM models is the efficient solution for discount bond prices $P(\tau)$ of all maturities $\tau$ at each time $t$ of a 3-dimensional parabolic partial differential equation (PDE) of the form

$$\frac{\partial P_t(\tau)}{\partial \tau} = \sum_{i,j=1}^{3} a_{ij} \frac{\partial^2 P_t(\tau)}{\partial y_i \partial y_j} + \sum_{i=1}^{3} b_i \frac{\partial P_t(\tau)}{\partial y_i} + cP_t(\tau)$$

- Kim and Singleton’s 2-dimensional alternating direction implicit (ADI) solution method will not cope with the 3-D case.


- Having evaluated simulation-based techniques we intend to investigate applying a fast robust 3D PDE solver based on an interpolating wavelet-specified irregular mesh implicit method that we have developed for complex derivative valuation and is expected to form a part of the NAG library.

Black Model Calibration Progress

- **Bonfim (2003)** estimated his 2 factor model only on yields safely above 0 where the underlying shadow rate affine model rates and the Black rates agree, used the standard KF in the EM algorithm and solved the 2D parabolic quasilinear bond price PDE with finite differences.

- **Bauer & Rudebusch (2014)** took the same approach to the 3 factor model employing the EKF in the EM algorithm and evaluated bond prices using 500 path Monte Carlo simulation as do **Lemke & Vladu (2014)** with more paths.

- **Dempster et al. (2014)** used least squares with 4 observed yields, QMLE and analytical approximation for short yields and 10,000 path Monte Carlo for longer maturity yields as noted above.
Further Progress

- Krippner (2013) noted that Black model bond prices for short maturities $\tau$ at time $t$ are the underlying shadow rate model closed form bond price less the value of an American call option struck at $t$ with strike 1 and maturity $\tau$ which he approximates with the corresponding Black Scholes European value but although this is accurate under P it is not under Q and hence is not arbitrage free. These prices can however be used as control variates with Monte Carlo bond price evaluation.

- Richard (2013) solves the 3D bond price PDE for the Black model using finite differences in 2 to 4 weeks on a supercomputer!

- Priebsch (2013) notes that the Black log bond price is the value at -1 of the conditional cumulant generating function of the random integral term involved under Q which can be expanded as

$$\ln P_t(\tau) = \ln E^Q_t \left[ \exp \left( \int_t^{t+\tau} 0 \sigma s \ ds \right) \right] = \sum_{j=1}^{\infty} (-1)^j \frac{\kappa^Q_j}{j!} \quad \text{and then 1 or 2 terms used}$$

- We apply the Black correction to the measurement equation for yields within the unscented Kalman filter together with QMLE in the EM algorithm and EFM bond prices.
Unscented Kalman Filter Bond Pricing

- Here we calibrate the Black EFM model with our current EM algorithm approach using the (NAG) unscented Kalman filter to handle the “hockey stick” nonlinearity. JULIER & UHLMANN (1997)
- Working with yields directly as we do rather than bond prices computed or approximated numerically from integrals of the instantaneous short rate as in the references to Black model calibration previously cited significantly accelerates computation.
- Putting the EFM 3-factor yield curve dynamics in state-space form shows that the factor state dynamics remain linear Gaussian while the Black nonlinearity may be directly applied to each observed maturity market rate in the shadow rate affine measurement equation – longer maturity yields typically need no correction.
- With this approach the 35 (34 sigma points plus original) duplicate KF calculations of the unscented Kalman filter averaged at each daily time step can be mindlessly parallelized to handle the Black nonlinearity in essentially the same running time as the calibration of the underlying EFM model using basic linear Kalman filtering.
Unscented Kalman Filter Implementation
Parallelization Schema with MPI

Master thread
DIRECT, Powell optimization, slaves synchronizing

Slave thread 1
Kalman filter

Slave thread 2
Kalman filter

... ...

Slave thread 30
Kalman filter

Slave thread 31
Kalman filter

Shared space with common variables: objective function value, predictions
Data

- Combination of LIBOR data and fixed interest rate swap rates (the ISDA fix) for each of 4 currency areas (EUR, GBP, USD, JPY) to bootstrap the yield curve daily for 14 maturities:
  
  3 month, 6 month, 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, 7 years, 8 years, 9 years, 10 years, 20 years, 30 years

- In the case of the Swiss franc (CHF), only 12 maturities are available:
  
  3 month, 6 month, 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, 7 years, 8 years, 9 years, 10 years

- Calibration periods used for these 5 currencies are the following:
  
  EUR: 02.01.2001 to 02.01.2012
  CHF: 02.01.2001 to 31.05.2013
  GBP: 07.10.2008 to 31.05.2013
  USD: 02.01.2001 to 31.05.2013
  JPY: 30.03.2009 to 31.05.2013

- The data was obtained from Bloomberg
In-sample Yield Curve Goodness-of-fit

EUR Date: 22 Aug 2008

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFM</td>
<td>11 bp</td>
</tr>
<tr>
<td>Black EFM</td>
<td>6 bp</td>
</tr>
</tbody>
</table>
CHF  Date: 20 Aug 2001

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFM</td>
<td>2 bp</td>
</tr>
<tr>
<td>Black EFM</td>
<td>2 bp</td>
</tr>
</tbody>
</table>

© 2015 Cambridge Systems Associates Limited
www.cambridge-systems.com
GBP  Date: 18 Feb 2013

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFM</td>
<td>8 bp</td>
</tr>
<tr>
<td>Black EFM</td>
<td>5 bp</td>
</tr>
</tbody>
</table>

© 2015 Cambridge Systems Associates Limited
www.cambridge-systems.com
USD  Date: 14 Oct 2008

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFM</td>
<td>14 bp</td>
</tr>
<tr>
<td>Black EFM</td>
<td>5 bp</td>
</tr>
<tr>
<td>Model</td>
<td>RMSE</td>
</tr>
<tr>
<td>------------</td>
<td>------</td>
</tr>
<tr>
<td>EFM</td>
<td>16 bp</td>
</tr>
<tr>
<td>Black EFM</td>
<td>4 bp</td>
</tr>
</tbody>
</table>

JPY

Date: 12 Nov 2012
## Overall In-sample Goodness of Fit

<table>
<thead>
<tr>
<th>Currency</th>
<th>Observations</th>
<th>Calibration</th>
<th>log likelihood</th>
<th>Sample fit MSE (vol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>2817</td>
<td>EFM</td>
<td>232,652</td>
<td>15 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EFM UKF</td>
<td>252,500</td>
<td>17 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Black EFM $\alpha=0.0025$</td>
<td>259,436</td>
<td>15 bp</td>
</tr>
<tr>
<td>CHF</td>
<td>3100</td>
<td>EFM</td>
<td>232,100</td>
<td>8 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EFM UKF</td>
<td>250,391</td>
<td>10 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Black EFM $\alpha=1.0$</td>
<td>253,095</td>
<td>8 bp</td>
</tr>
<tr>
<td>GBP</td>
<td>1171</td>
<td>EFM</td>
<td>98,021</td>
<td>16 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EFM UKF</td>
<td>103,529</td>
<td>15 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Black EFM $\alpha=0.0001$</td>
<td>105,368</td>
<td>14 bp</td>
</tr>
<tr>
<td>USD</td>
<td>3093</td>
<td>EFM</td>
<td>279,114</td>
<td>15 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EFM UKF</td>
<td>280,745</td>
<td>25 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Black EFM $\alpha=0.0001$</td>
<td>288,422</td>
<td>16 bp</td>
</tr>
<tr>
<td>JPY</td>
<td>950</td>
<td>EFM</td>
<td>91,014</td>
<td>6 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EFM UKF</td>
<td>84,564</td>
<td>28 bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Black EFM $\alpha=0.006$</td>
<td>102,544</td>
<td>6 bp</td>
</tr>
</tbody>
</table>
Monte Carlo Out-of-sample 30 Year Projection

Quantiles based on 100,000 scenarios

30 Year EFM GPB 10 Year Rate
Monte Carlo Out-of-sample Projection

Quantiles based on 100,000 scenarios

30 Year Black EFM GPB 10 Year Rate

GBP 10 year rate forecast RMSE over 20 months

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Black median</td>
<td>0.48%</td>
</tr>
<tr>
<td>EFM median</td>
<td>0.45%</td>
</tr>
</tbody>
</table>

GBP 10 year rate Black UKF alpha = 0.0001
EUR 10 Year Rate Out-of-sample Projections

EUR 10 year rate EFM

EUR 10 year rate Black UKF alpha = 0.0025

EUR 10 year rate forecast RMSE over 37 months

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Black median</td>
<td>1.76%</td>
</tr>
<tr>
<td>EFM median</td>
<td>1.37%</td>
</tr>
</tbody>
</table>
CHF 10 Year Rate Out-of-sample Projections

CHF 10 year rate EFM

CHF 10 year Black UKF alpha = 1.0

CHF 10 year rate forecast RMSE over 20 months

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Black median</td>
<td>0.40%</td>
</tr>
<tr>
<td>EFM median</td>
<td>0.45%</td>
</tr>
</tbody>
</table>
USD 10 Year Rate Out-of-sample Projections

USD 10 year rate forecast RMSE over 21 months

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black median</td>
<td>0.39%</td>
</tr>
<tr>
<td>EFM median</td>
<td>0.43%</td>
</tr>
</tbody>
</table>
JPY 10 Year Rate Out-of-sample Projections
Conclusion
Conclusions

- We have developed a **Black-corrected version** of our workhorse 3 factor affine Gaussian yield curve **Economic Factor Model** implemented using the unscented Kalman filter to handle the Black nonlinearity and **HPC** techniques.

- Although this method generates an **approximation** to the full Black model its accuracy is comparable to and computing run time only about twice that of the basic EFM model – unlike all the alternatives published to date which are very heavily computationally intensive.

- Using the **unreleased NAG UKF algorithm (g13 ejc)** with tuned $\alpha$ parameter setting both the in- and out-of-sample accuracy of the method **exceeds** that of the affine EFM model and it possesses much better dynamics.

- Using the **cloud** we can reduce **calibration times on big samples** to minutes.
References


